



# Holiday Homework CLASS –X

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## Subject English

### SUBJECT ENRICHMENT ACTIVITIES

(in subject enrichment copy for the written work)

1. Write two short stories of your own
2. Write two poems that you like the most
3. Write an experience of your visit to a place
4. Write ten amazing facts about OTTER
5. Paste/draw the picture of A TIGER IN THE FOREST AND A TIGER IN A ZOO, write an article on " Freedom is the birth right of every living being"
6. Write the THREE VERB FORMS of IRREGULAR VERBS
7. Read English newspapers, magazines, comics, story books every day.
8. Listen to English news for at least ten to twenty minutes.
9. WRITE THE BIOSKETCH OF NELSON MANDELA

#### IN HOMEWORK COPY

1. Which part of the story, "THE TRIUMPH OF SURGERY " do you like the most and why?
2. Write a paragraph on "FIRM DETERMINATION AND STRONG WILLPOWER CAN ACHIEVE EVEN THE IMPOSSIBLE "
3. Write an article on "NEW EDUCATION SYSTEM OF INDIA"
4. Write the best ten quotes of DR.APJ ABDUL KALAM
5. Write ten amazing POINTS OF "THE FREEDOM STRUGGLE OF SOUTH AFRICA"  
# The students must visit a post office, write their observations as a diary entry.  
# THE STUDENTS WHO HAVE PARTICIPATED IN "YUVA BHAGIDARI - G20" VISITED IIT/KIIT/IIMT, HAVE TO PREPARE THE DOCUMENT IN 5000 WORDS

## विषय – हिंदी कक्षा – दसवीं

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1. पाठ्यक्रम की कोई भी एक कविता याद करके तथा उसका वीडियो बनाकर मुझे व्यक्तिगत प्रेषित करें ।

(कविता डेढ़ मिनट से पाँच मिनट ) (वाचन-कौशल)

2. 'बुरांश शीतल पेय' पर या अपने द्वारा बनाई गई 'गोपीनाथ अगरबत्ती' ' पर विज्ञापन तैयार करें । (60 शब्द)

3. फुटबॉल मैच जीतने पर अपने मित्र को उसे बधाई संदेश प्रेषित करें। (60 शब्द)

4. 'पहाड़ों का महत्त्व' या 'आज तकनीकी का ज्ञान कितना आवश्यक' विषय पर अनुच्छेद लिखिए ।

5. निम्नलिखित में से कोई दो फ़िल्में देखिए और उनसे मिलने वाली पाँच प्रमुख शिक्षाएँ लिखिए :—

i :- <https://youtu.be/gZy4vIGf7MY>

I am kalam

ii :- <https://youtu.be/CPXkijYI9Y0>

Chalk n duster

iii :- <https://youtu.be/a1G1Sg3-g2g>

Taare zameen par

iv :- <https://youtu.be/l3Ssqdk88gH4>

Baghban

v :- <https://youtu.be/epKzi21TRN8>

उम्मीद (लघु फ़िल्म )

vi :- <https://youtu.be/OnhZDZXzBz4>

रद्दी लाइब्रेरी (लघु फ़िल्म )

6. पोर्टफोलिओ बनाएँ जो कि आपको साथ ही भेजा जा रहा है। (A4 साइज़ पेपर में)

7. ओड़िसा के किन्हीं दस स्वतंत्रता संग्राम सेनानियों की परियोजना फ़ाइल बनाएँ, जिसमें इन सेनानियों की फ़ोटो, नाम, जन्म-मृत्यु तिथि, महत्त्वपूर्ण कार्य/ स्वतंत्रता प्राप्ति हेतु सहयोग आदि का वर्णन हो। (A4 साइज़ पेपर में)

8. अभी तक जो भी पढ़ाया गया है, उसको कम-से कम पाँच बार दोहराएँ। आते ही उसी से प्रश्न पूछे जाएँगे।

9. कटक ज़िले के दर्शनीय स्थलों की जानकारी देते हुए मुम्बई -स्थित अपने मित्र को पत्र लिखिए । (100 शब्द)

10. आपको किसी विद्यालय में हिंदी-अध्यापक की नौकरी पानी है। अतः विद्यालय के निदेशक को

नौकरी हेतु आवेदन-पत्र लिखें। इसमें आपका स्ववृत्त (biodata) होना अति आवश्यक है। (80 शब्द)

11. किसी भी कम्पनी के सीईओ को औपचारिक ई-मेल भेजें, इसका विषय आप कुछ भी ले सकते हैं।(80 शब्द)

12. कला समेकित परियोजना :- ओड़िसा का जनजीवन, भाषाओं पर पीपीटी व वीडियो (Art Integrated Project)

नोट :-छुट्टियों के शुरू के चार दिनों में ही गृहकार्य करके आप मुझे उसकी फोटो लेकर भेज दीजिएगा ।

## **SUBJECT: SANSKRIT**

### **KENDRIYA VIDYALAYA NO.2 CUTTACK**

#### **ग्रीष्मकालीन अवकाश गृह कार्य**

**कक्षा - दशमी**

**विषय: - संस्कृत**

1.मातुः अस्वस्थता विषये प्राचार्यं प्रति एकं पत्रं लिखत।

2.स्था , हस् धातुः च पञ्चलकारानां रूपं लिखत।( लट्, लृट्, लोट्, लङ्, च विधिलिङ्ग)

3.संधि विच्छेदं कुरुत -

विद्यालयं, सदैव, परोपकार, गिरीश, स्वागतम्।

4. पर्यावरणस्य महत्त्वस्य विषये संस्कृते निबंधः लिखत ।

5. "यत्", "एतत्" च ' शब्दरूपाणि लिखत।

6. कस्य द्वयोः विषयोः चित्रवर्णनं कुरुत-

1.पुस्तकालयस्य चित्रं 2. विद्यालयस्य चित्रं 3. रक्षाबंधनपर्वस्य चित्रं

7. अधोलिखितं अनुच्छेदं पठित्वा प्रश्नान् उत्तरत - एकस्मिन् नगरे द्वे मित्रे वसतः स्म । एकस्य नाम सोमेशः आसीत् । अन्यस्य नाम धनेशः आसीत् । सोमेशः विद्यां इच्छति स्म । धनेशः प्रभूतं धनं इच्छति स्म । एकदा मित्रद्वयं विदेशं अगच्छत् । तत्र सोमेशः परिश्रमेण अध्ययनं कृत्वा विद्यां प्राप्तवान् । धनेशः बहु- धनसङ्ग्रहं कृतवान् । एवं अनेकानि वर्षाणि व्यतीतानि । तौ अचिन्तयतां - 'अधुना आवां गृहं गच्छावः' । गृहं प्रति आगमनसमये मार्गं चौराः आगच्छन् । ते धनेशस्य सर्वं धनं अहरन् । धनेशः दुःखी अभवत् । सः रिक्तहस्तः गृहं आगच्छत् । परन्तु सोमेशः विद्याधनयुक्तः आसीत् । विद्याधनेन युक्तः सः शीघ्रमेव अतीप्रसिद्धः अभवत् । राजा तस्य संमानं कृत्वा मन्त्रिपदं अपि अयच्छत् । अत एव उच्यते - " विद्या एव सर्वत्र पूज्यते । "

(I) एकपदेन उत्तरत - 1. एकस्मिन् नगरे के वसतः स्म?

2. कः प्रभूतं धनं इच्छति स्म?

(II) पूर्णवाक्येन उत्तरत- 1. गृहं प्रति आगमनसमये मार्गं के आगच्छन्?

2. विद्याधनेन युक्तः सः शीघ्रमेव कीदृशः अभवत्?

(III) निर्देशानुसारं उत्तरत -1." अप्रसिद्धः " इति पदस्य विलोमपदं किं प्रयुक्तम् ?

2." बहुधनम् " इति अर्थे किं पदं प्रयुक्तम् ?

## Summer Holiday Homework

Class X

Subject- Science

### 1. Exemplar Questions:

Complete the NCERT exemplar Questions of chapter- Chemical equation and reaction.

### 2. Mind map:

Prepare the mind map of

- a. Types of nutrition
- b. Types of equations.

### 3. Assortion- Reason questions:

Following questions consist of two statements – Assertion (A) and Reason (R). Answer these questions selecting the appropriate option given below:

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

Q.1. Assertion (A) : Decomposition of vegetable matter into compost is an example of exothermic reactions.

Reason (R) : Exothermic reaction are those reactions in which heat is evolved.

Q.2. Assertion (A) : When HCl is added to zinc granules, a chemical reaction occurs.

Reason (R) : Evolution of a gas and change in colour indicate that the chemical reaction is taking place.

Q.3. Assertion (A) : Calcium carbonate when heated gives calcium oxide and water.

Reason (R) : On heating calcium carbonate, decomposition reaction takes place.

Q.4. Assertion (A) : Brown fumes are produced when lead nitrate is heated.

Reason (R) : Nitrogen dioxide gas is produced as a by product due to the decomposition of lead



nitrate.

Q.5. Assertion (A) : White silver chloride turns grey in sunlight.

Reason (R) : Decomposition of silver chloride in presence of sunlight takes place to form silver metal and chlorine gas.

Q.6. Assertion (A): Pungent smelling gas is produced when sulphur burns in air.

Reason (R) : Sulphur trioxide is formed on reaction of sulphur with oxygen.

Q.7. Assertion (A) : In a reaction of copper with oxygen, copper serves as a reducing agent.

Reason (R) : The substance which gains oxygen in a chemical reaction acts as a reducing agent.

Q.8. Assertion (A) : In electrolysis of water, the volume of hydrogen liberated is twice the volume of oxygen formed.

Reason (R) : Water (H<sub>2</sub>O) has hydrogen and oxygen in the ratio of 1:2 by volume.

Q.9. Assertion (A): Corrosion of iron is commonly known as rusting.

Reason (R) : Corrosion of iron occurs in presence of water and air.

Q.10. Assertion (A) : The balancing of chemical equations is based on law of conservation of mass.

Reason (R) : Total mass of reactants is equal to total mass of products.

#### 4. Balancing equations:

Take 15 skeleton equations, and solve those in both Hit and trial and ABCD method.

#### 5. Competency based questions:

complete the test items( competency based questions) sent in whats app group (Only write the answers)

#### 6. Minimum level of learning for Class X

Complete and practice the questions given in whats app.



## Subject - Social science

Q1 Make a project file on the topic consumer's rights.

Q2 Prepare 10 -10 MCQs from each lesson;

1) NATIONALISM IN EUROPE (HISTORY)

2) RESOURCES AND DEVELOPMENT (GEOGRAPHY)

Q3. On an outline map of India locate the following:

A. Nagpur, Kolkata, Dandi, Lucknow, Chauri Choura, Champaran, Kheda, Ahmedabad

C. Soil in different regions.

D. Sea ports and airports

Q4. How did culture play an important role in creating the idea of the Nation in Europe. explain .

Q5. explain about Unification of Italy?

## SUBJECT: ART

1. Make any five 3D craft work

Reference :

<https://www.youtube.com/shorts/P2yhWk51zXI>  
[youtube.com/watch?v=pDkUlrJFVaE](https://www.youtube.com/watch?v=pDkUlrJFVaE)

2. Draw any five portraits ,2D (colour or shading)

Reference :

<https://www.youtube.com/shorts/FndjSSmij3s>  
<https://www.youtube.com/shorts/WNEgdUxlX04>

## SUBJECT : WORK EDUCATION

KENDRIYA VIDYALAYA NO.2 CUTTACK (2023-2024)  
HOLIDAY HOME ASSIGNMENTS FOR SUMMER VACATION

SL NO	CLASS	TOPIC/THEME	ASSIGNMENT	TLO/Skill
01	X	Cover design, Best out of waste, origami(geometrical) Electrical Symbols	Origami work(04) Best out of waste(04) Chart for Symbols	To make various models Awareness Creativity

## **SUBJECT: AI**

- A.** Prepare a 1000 words write-up or a 5 slide presentation on **How insects Inspire ArtificialIntelligence.**
  - B.** Complete the exercise of chapter self management skill
- 



**Submit homework on very first day after reopening of Vidyalaya**

आप सभी स्वस्थ रहें , मस्त रहें , खुश रहें , सीखते रहें , पेरेंट्स की सहायता करते रहें ।

बहुत बहुत

आशीर्वाद सभी विद्यार्थियों को।

-----Wish you all a very happy vacation, take care-----

## REAL NUMBERS

### (A) Main Concepts and Results

- Euclid's Division Lemma : Given two positive integers  $a$  and  $b$ , there exist unique integers  $q$  and  $r$  satisfying  $a = bq + r$ ,  $0 \leq r < b$ .
- Euclid's Division Algorithm to obtain the HCF of two positive integers, say  $c$  and  $d$ ,  $c > d$ .

**Step 1 :** Apply Euclid's division lemma to  $c$  and  $d$ , to find whole numbers  $q$  and  $r$ , such that  $c = dq + r$ ,  $0 \leq r < d$ .

**Step 2 :** If  $r = 0$ ,  $d$  is the HCF of  $c$  and  $d$ . If  $r \neq 0$ , apply the division lemma to  $d$  and  $r$ .

**Step 3 :** Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

- Fundamental Theorem of Arithmetic : Every composite number can be expressed as a product of primes, and this expression (factorisation) is unique, apart from the order in which the prime factors occur.
- Let  $p$  be a prime number. If  $p$  divides  $a^2$ , then  $p$  divides  $a$ , where  $a$  is a positive integer.
- $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$  are irrational numbers.
- The sum or difference of a rational and an irrational number is irrational.
- The product or quotient of a non-zero rational number and an irrational number is irrational.
- For any two positive integers  $a$  and  $b$ ,  $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$ .



- Let  $x = \frac{p}{q}$ ,  $p$  and  $q$  are co-prime, be a rational number whose decimal expansion terminates. Then, the prime factorisation of  $q$  is of the form  $2^m \cdot 5^n$ ;  $m, n$  are non-negative integers.
- Let  $x = \frac{p}{q}$  be a rational number such that the prime factorisation of  $q$  is not of the form  $2^m \cdot 5^n$ ;  $m, n$  being non-negative integers. Then,  $x$  has a non-terminating repeating decimal expansion.

### (B) Multiple Choice Questions

Choose the correct answer from the given four options:

**Sample Question 1 :** The decimal expansion of the rational number  $\frac{33}{2^2 \cdot 5}$  will terminate after

- (A) one decimal place                      (B) two decimal places  
(C) three decimal places                  (D) more than 3 decimal places

**Solution :** Answer (B)

**Sample Question 2 :** Euclid's division lemma states that for two positive integers  $a$  and  $b$ , there exist unique integers  $q$  and  $r$  such that  $a = bq + r$ , where  $r$  must satisfy

- (A)  $1 < r < b$                                   (B)  $0 < r \leq b$   
(C)  $0 \leq r < b$                                   (D)  $0 < r < b$

**Solution :** Answer (C)

### EXERCISE 1.1

Choose the correct answer from the given four options in the following questions:

- For some integer  $m$ , every even integer is of the form
 

(A)  $m$                                               (B)  $m + 1$   
(C)  $2m$                                               (D)  $2m + 1$
- For some integer  $q$ , every odd integer is of the form
 

(A)  $q$                                                 (B)  $q + 1$   
(C)  $2q$                                                 (D)  $2q + 1$

3.  $n^2 - 1$  is divisible by 8, if  $n$  is  
(A) an integer (B) a natural number  
(C) an odd integer (D) an even integer
4. If the HCF of 65 and 117 is expressible in the form  $65m - 117$ , then the value of  $m$  is  
(A) 4 (B) 2  
(C) 1 (D) 3
5. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is  
(A) 13 (B) 65  
(C) 875 (D) 1750
6. If two positive integers  $a$  and  $b$  are written as  
 $a = x^3y^2$  and  $b = xy^3$ ;  $x, y$  are prime numbers, then HCF ( $a, b$ ) is  
(A)  $xy$  (B)  $xy^2$  (C)  $x^3y^3$  (D)  $x^2y^2$
7. If two positive integers  $p$  and  $q$  can be expressed as  
 $p = ab^2$  and  $q = a^3b$ ;  $a, b$  being prime numbers, then LCM ( $p, q$ ) is  
(A)  $ab$  (B)  $a^2b^2$  (C)  $a^3b^2$  (D)  $a^3b^3$
8. The product of a non-zero rational and an irrational number is  
(A) always irrational (B) always rational  
(C) rational or irrational (D) one
9. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is  
(A) 10 (B) 100 (C) 504 (D) 2520
10. The decimal expansion of the rational number  $\frac{14587}{1250}$  will terminate after:  
(A) one decimal place (B) two decimal places  
(C) three decimal places (D) four decimal places

### (C) Short Answer Questions with Reasoning

**Sample Question 1:** The values of the remainder  $r$ , when a positive integer  $a$  is divided by 3 are 0 and 1 only. Justify your answer.

**Solution :** No.

According to Euclid's division lemma,

$$a = 3q + r, \text{ where } 0 \leq r < 3$$

and  $r$  is an integer. Therefore, the values of  $r$  can be 0, 1 or 2.

**Sample Question 2:** Can the number  $6^n$ ,  $n$  being a natural number, end with the digit 5? Give reasons.

**Solution :** No, because  $6^n = (2 \times 3)^n = 2^n \times 3^n$ , so the only primes in the factorisation of  $6^n$  are 2 and 3, and not 5.

Hence, it cannot end with the digit 5.

### EXERCISE 1.2

- Write whether every positive integer can be of the form  $4q + 2$ , where  $q$  is an integer. Justify your answer.
- "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.
- "The product of three consecutive positive integers is divisible by 6". Is this statement true or false"? Justify your answer.
- Write whether the square of any positive integer can be of the form  $3m + 2$ , where  $m$  is a natural number. Justify your answer.
- A positive integer is of the form  $3q + 1$ ,  $q$  being a natural number. Can you write its square in any form other than  $3m + 1$ , i.e.,  $3m$  or  $3m + 2$  for some integer  $m$ ? Justify your answer.
- The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer.
- Explain why  $3 \times 5 \times 7 + 7$  is a composite number.
- Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.
- Without actually performing the long division, find if  $\frac{987}{10500}$  will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.
- A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of  $q$ , when this number is expressed in the form  $\frac{p}{q}$ ? Give reasons.

**(D) Short Answer Questions**

**Sample Question 1:** Using Euclid's division algorithm, find which of the following pairs of numbers are co-prime:

- (i) 231, 396      (ii) 847, 2160

**Solution :** Let us find the HCF of each pair of numbers.

$$\begin{aligned} \text{(i)} \quad 396 &= 231 \times 1 + 165 \\ 231 &= 165 \times 1 + 66 \\ 165 &= 66 \times 2 + 33 \\ 66 &= 33 \times 2 + 0 \end{aligned}$$

Therefore, HCF = 33. Hence, numbers are not co-prime.

$$\begin{aligned} \text{(ii)} \quad 2160 &= 847 \times 2 + 466 \\ 847 &= 466 \times 1 + 381 \\ 466 &= 381 \times 1 + 85 \\ 381 &= 85 \times 4 + 41 \\ 85 &= 41 \times 2 + 3 \\ 41 &= 3 \times 13 + 2 \\ 3 &= 2 \times 1 + 1 \\ 2 &= 1 \times 2 + 0 \end{aligned}$$

Therefore, the HCF = 1. Hence, the numbers are co-prime.

**Sample Question 2:** Show that the square of an odd positive integer is of the form  $8m + 1$ , for some whole number  $m$ .

**Solution:** Any positive odd integer is of the form  $2q + 1$ , where  $q$  is a whole number.

$$\text{Therefore, } (2q + 1)^2 = 4q^2 + 4q + 1 = 4q(q + 1) + 1, \quad (1)$$

$q(q + 1)$  is either 0 or even. So, it is  $2m$ , where  $m$  is a whole number.

$$\text{Therefore, } (2q + 1)^2 = 4 \cdot 2m + 1 = 8m + 1. \quad [\text{From (1)}]$$

**Sample Question 3:** Prove that  $\sqrt{2} + \sqrt{3}$  is irrational.

**Solution :** Let us suppose that  $\sqrt{2} + \sqrt{3}$  is rational. Let  $\sqrt{2} + \sqrt{3} = a$ , where  $a$  is rational.

Therefore,  $\sqrt{2} = a - \sqrt{3}$

Squaring on both sides, we get

$$2 = a^2 + 3 - 2a\sqrt{3}$$

Therefore,  $\sqrt{3} = \frac{a^2 + 1}{2a}$ , which is a contradiction as the right hand side is a rational number while  $\sqrt{3}$  is irrational. Hence,  $\sqrt{2} + \sqrt{3}$  is irrational.

### EXERCISE 1.3

1. Show that the square of any positive integer is either of the form  $4q$  or  $4q + 1$  for some integer  $q$ .
2. Show that cube of any positive integer is of the form  $4m$ ,  $4m + 1$  or  $4m + 3$ , for some integer  $m$ .
3. Show that the square of any positive integer cannot be of the form  $5q + 2$  or  $5q + 3$  for any integer  $q$ .
4. Show that the square of any positive integer cannot be of the form  $6m + 2$  or  $6m + 5$  for any integer  $m$ .
5. Show that the square of any odd integer is of the form  $4q + 1$ , for some integer  $q$ .
6. If  $n$  is an odd integer, then show that  $n^2 - 1$  is divisible by 8.
7. Prove that if  $x$  and  $y$  are both odd positive integers, then  $x^2 + y^2$  is even but not divisible by 4.
8. Use Euclid's division algorithm to find the HCF of 441, 567, 693.
9. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3, respectively.
10. Prove that  $\sqrt{3} + \sqrt{5}$  is irrational.
11. Show that  $12^n$  cannot end with the digit 0 or 5 for any natural number  $n$ .
12. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

13. Write the denominator of the rational number  $\frac{257}{5000}$  in the form  $2^m \times 5^n$ , where  $m, n$  are non-negative integers. Hence, write its decimal expansion, without actual division.
14. Prove that  $\sqrt{p} + \sqrt{q}$  is irrational, where  $p, q$  are primes.

### (E) Long Answer Questions

**Sample Question 1 :** Show that the square of an odd positive integer can be of the form  $6q + 1$  or  $6q + 3$  for some integer  $q$ .

**Solution :** We know that any positive integer can be of the form  $6m, 6m + 1, 6m + 2, 6m + 3, 6m + 4$  or  $6m + 5$ , for some integer  $m$ .

Thus, an odd positive integer can be of the form  $6m + 1, 6m + 3$ , or  $6m + 5$   
Thus we have:

$$(6m + 1)^2 = 36m^2 + 12m + 1 = 6(6m^2 + 2m) + 1 = 6q + 1, q \text{ is an integer}$$

$$(6m + 3)^2 = 36m^2 + 36m + 9 = 6(6m^2 + 6m + 1) + 3 = 6q + 3, q \text{ is an integer}$$

$$(6m + 5)^2 = 36m^2 + 60m + 25 = 6(6m^2 + 10m + 4) + 1 = 6q + 1, q \text{ is an integer.}$$

Thus, the square of an odd positive integer can be of the form  $6q + 1$  or  $6q + 3$ .

### EXERCISE 1.4

1. Show that the cube of a positive integer of the form  $6q + r$ ,  $q$  is an integer and  $r = 0, 1, 2, 3, 4, 5$  is also of the form  $6m + r$ .
2. Prove that one and only one out of  $n, n + 2$  and  $n + 4$  is divisible by 3, where  $n$  is any positive integer.
3. Prove that one of any three consecutive positive integers must be divisible by 3.
4. For any positive integer  $n$ , prove that  $n^3 - n$  is divisible by 6.
5. Show that one and only one out of  $n, n + 4, n + 8, n + 12$  and  $n + 16$  is divisible by 5, where  $n$  is any positive integer.

[Hint: Any positive integer can be written in the form  $5q, 5q+1, 5q+2, 5q+3, 5q+4$ ].

## POLYNOMIALS

### (A) Main Concepts and Results

- Geometrical meaning of zeroes of a polynomial: The zeroes of a polynomial  $p(x)$  are precisely the  $x$ -coordinates of the points where the graph of  $y = p(x)$  intersects the  $x$ -axis.
- Relation between the zeroes and coefficients of a polynomial: If  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial  $ax^2 + bx + c$ , then  $\alpha + \beta = -\frac{b}{a}$ ,  $\alpha\beta = \frac{c}{a}$ .
- If  $\alpha$ ,  $\beta$  and  $\gamma$  are the zeroes of a cubic polynomial  $ax^3 + bx^2 + cx + d$ , then  $\alpha + \beta + \gamma = -\frac{b}{a}$ ,  $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$  and  $\alpha\beta\gamma = \frac{-d}{a}$ .
- The division algorithm states that given any polynomial  $p(x)$  and any non-zero polynomial  $g(x)$ , there are polynomials  $q(x)$  and  $r(x)$  such that  $p(x) = g(x)q(x) + r(x)$ , where  $r(x) = 0$  or degree  $r(x) <$  degree  $g(x)$ .

### (B) Multiple Choice Questions

Choose the correct answer from the given four options:

**Sample Question 1:** If one zero of the quadratic polynomial  $x^2 + 3x + k$  is 2, then the value of  $k$  is

- (A) 10                      (B) -10                      (C) 5                      (D) -5

**Solution :** Answer (B)

**Sample Question 2:** Given that two of the zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$  are 0, the third zero is

- (A)  $\frac{-b}{a}$       (B)  $\frac{b}{a}$       (C)  $\frac{c}{a}$       (D)  $-\frac{d}{a}$

**Solution :** Answer (A). [**Hint:** Because if third zero is  $\alpha$ , sum of the zeroes

$$= \alpha + 0 + 0 = \frac{-b}{a} ]$$

### EXERCISE 2.1

Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial  $(k-1)x^2 + kx + 1$  is  $-3$ , then the value of  $k$  is

- (A)  $\frac{4}{3}$       (B)  $\frac{-4}{3}$       (C)  $\frac{2}{3}$       (D)  $\frac{-2}{3}$

2. A quadratic polynomial, whose zeroes are  $-3$  and  $4$ , is

- (A)  $x^2 - x + 12$       (B)  $x^2 + x + 12$   
 (C)  $\frac{x^2}{2} - \frac{x}{2} - 6$       (D)  $2x^2 + 2x - 24$

3. If the zeroes of the quadratic polynomial  $x^2 + (a+1)x + b$  are  $2$  and  $-3$ , then

- (A)  $a = -7, b = -1$       (B)  $a = 5, b = -1$   
 (C)  $a = 2, b = -6$       (D)  $a = 0, b = -6$

4. The number of polynomials having zeroes as  $-2$  and  $5$  is

- (A) 1      (B) 2      (C) 3      (D) more than 3

5. Given that one of the zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$  is zero, the product of the other two zeroes is

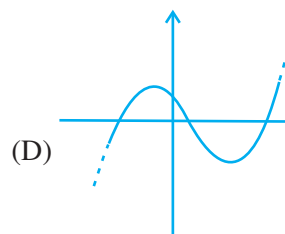
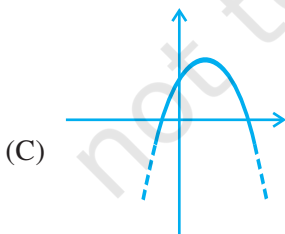
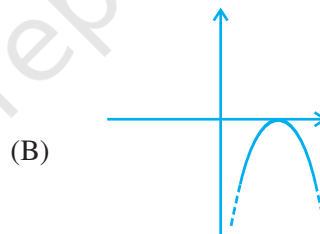
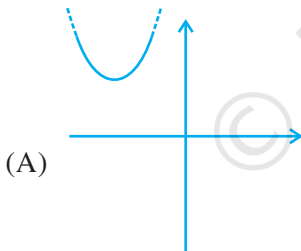
- (A)  $-\frac{c}{a}$       (B)  $\frac{c}{a}$       (C) 0      (D)  $-\frac{b}{a}$

6. If one of the zeroes of the cubic polynomial  $x^3 + ax^2 + bx + c$  is  $-1$ , then the product of the other two zeroes is

- (A)  $b - a + 1$       (B)  $b - a - 1$       (C)  $a - b + 1$       (D)  $a - b - 1$



7. The zeroes of the quadratic polynomial  $x^2 + 99x + 127$  are  
 (A) both positive (B) both negative  
 (C) one positive and one negative (D) both equal
8. The zeroes of the quadratic polynomial  $x^2 + kx + k, k \neq 0$ ,  
 (A) cannot both be positive (B) cannot both be negative  
 (C) are always unequal (D) are always equal
9. If the zeroes of the quadratic polynomial  $ax^2 + bx + c, c \neq 0$  are equal, then  
 (A)  $c$  and  $a$  have opposite signs (B)  $c$  and  $b$  have opposite signs  
 (C)  $c$  and  $a$  have the same sign (D)  $c$  and  $b$  have the same sign
10. If one of the zeroes of a quadratic polynomial of the form  $x^2 + ax + b$  is the negative of the other, then it  
 (A) has no linear term and the constant term is negative.  
 (B) has no linear term and the constant term is positive.  
 (C) can have a linear term but the constant term is negative.  
 (D) can have a linear term but the constant term is positive.
11. Which of the following is not the graph of a quadratic polynomial?



**(C) Short Answer Questions with Reasoning**

**Sample Question 1:** Can  $x - 1$  be the remainder on division of a polynomial  $p(x)$  by  $2x + 3$ ? Justify your answer.

**Solution :** No, since degree  $(x - 1) = 1 = \text{degree}(2x + 3)$ .

**Sample Question 2:** Is the following statement True or False? Justify your answer.

If the zeroes of a quadratic polynomial  $ax^2 + bx + c$  are both negative, then  $a$ ,  $b$  and  $c$  all have the same sign.

**Solution :** True, because  $-\frac{b}{a} = \text{sum of the zeroes} < 0$ , so that  $\frac{b}{a} > 0$ . Also the product

of the zeroes  $= \frac{c}{a} > 0$ .

**EXERCISE 2.2**

1. Answer the following and justify:
  - (i) Can  $x^2 - 1$  be the quotient on division of  $x^6 + 2x^3 + x - 1$  by a polynomial in  $x$  of degree 5?
  - (ii) What will the quotient and remainder be on division of  $ax^2 + bx + c$  by  $px^3 + qx^2 + rx + s$ ,  $p \neq 0$ ?
  - (iii) If on division of a polynomial  $p(x)$  by a polynomial  $g(x)$ , the quotient is zero, what is the relation between the degrees of  $p(x)$  and  $g(x)$ ?
  - (iv) If on division of a non-zero polynomial  $p(x)$  by a polynomial  $g(x)$ , the remainder is zero, what is the relation between the degrees of  $p(x)$  and  $g(x)$ ?
  - (v) Can the quadratic polynomial  $x^2 + kx + k$  have equal zeroes for some odd integer  $k > 1$ ?
2. Are the following statements 'True' or 'False'? Justify your answers.
  - (i) If the zeroes of a quadratic polynomial  $ax^2 + bx + c$  are both positive, then  $a$ ,  $b$  and  $c$  all have the same sign.
  - (ii) If the graph of a polynomial intersects the  $x$ -axis at only one point, it cannot be a quadratic polynomial.
  - (iii) If the graph of a polynomial intersects the  $x$ -axis at exactly two points, it need not be a quadratic polynomial.
  - (iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

- (v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.
- (vi) If all three zeroes of a cubic polynomial  $x^3 + ax^2 - bx + c$  are positive, then at least one of  $a$ ,  $b$  and  $c$  is non-negative.
- (vii) The only value of  $k$  for which the quadratic polynomial  $kx^2 + x + k$  has equal zeros is  $\frac{1}{2}$

### (D) Short Answer Questions

**Sample Question 1:** Find the zeroes of the polynomial  $x^2 + \frac{1}{6}x - 2$ , and verify the relation between the coefficients and the zeroes of the polynomial.

**Solution :**  $x^2 + \frac{1}{6}x - 2 = \frac{1}{6} (6x^2 + x - 12) = \frac{1}{6} [6x^2 + 9x - 8x - 12]$

$$= \frac{1}{6} [3x(2x + 3) - 4(2x + 3)] = \frac{1}{6} (3x - 4)(2x + 3)$$

Hence,  $\frac{4}{3}$  and  $-\frac{3}{2}$  are the zeroes of the given polynomial.

The given polynomial is  $x^2 + \frac{1}{6}x - 2$ .

The sum of zeroes =  $\frac{4}{3} + -\frac{3}{2} = \frac{-1}{6} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$  and

the product of zeroes =  $\frac{4}{3} \times \frac{-3}{2} = -2 = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

### EXERCISE 2.3

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials:

1.  $4x^2 - 3x - 1$

2.  $3x^2 + 4x - 4$

3.  $5t^2 + 12t + 7$

4.  $t^3 - 2t^2 - 15t$

5.  $2x^2 + \frac{7}{2}x + \frac{3}{4}$

6.  $4x^2 + 5\sqrt{2}x - 3$

7.  $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

8.  $v^2 + 4\sqrt{3}v - 15$

9.  $y^2 + \frac{3}{2}\sqrt{5}y - 5$

10.  $7y^2 - \frac{11}{3}y - \frac{2}{3}$

**(E) Long Answer Questions**

**Sample Question 1:** Find a quadratic polynomial, the sum and product of whose zeroes are  $\sqrt{2}$  and  $-\frac{3}{2}$ , respectively. Also find its zeroes.

**Solution :** A quadratic polynomial, the sum and product of whose zeroes are

$$\sqrt{2} \text{ and } -\frac{3}{2} \text{ is } x^2 - \sqrt{2}x - \frac{3}{2}$$

$$\begin{aligned} x^2 - \sqrt{2}x - \frac{3}{2} &= \frac{1}{2} [2x^2 - 2\sqrt{2}x - 3] \\ &= \frac{1}{2} [2x^2 + \sqrt{2}x - 3\sqrt{2}x - 3] \\ &= \frac{1}{2} [\sqrt{2}x(\sqrt{2}x + 1) - 3(\sqrt{2}x + 1)] \\ &= \frac{1}{2} [\sqrt{2}x + 1][\sqrt{2}x - 3] \end{aligned}$$

Hence, the zeroes are  $-\frac{1}{\sqrt{2}}$  and  $\frac{3}{\sqrt{2}}$ .

**Sample Question 2:** If the remainder on division of  $x^3 + 2x^2 + kx + 3$  by  $x - 3$  is 21, find the quotient and the value of  $k$ . Hence, find the zeroes of the cubic polynomial  $x^3 + 2x^2 + kx - 18$ .

**Solution :** Let  $p(x) = x^3 + 2x^2 + kx + 3$

Then,  $p(3) = 3^3 + 2 \times 3^2 + 3k + 3 = 21$

i.e.,  $3k = -27$

i.e.,  $k = -9$

Hence, the given polynomial will become  $x^3 + 2x^2 - 9x + 3$ .

Now,  $(x - 3) x^3 + 2x^2 - 9x + 3(x^2 + 5x + 6)$

$$\begin{array}{r} x^3 - 3x^2 \\ \hline 5x^2 - 9x + 3 \\ 5x^2 - 15x \\ \hline 6x + 3 \\ 6x - 18 \\ \hline 21 \end{array}$$

So,  $x^3 + 2x^2 - 9x + 3 = (x^2 + 5x + 6)(x - 3) + 21$

i.e.,  $x^3 + 2x^2 - 9x - 18 = (x - 3)(x^2 + 5x + 6)$   
 $= (x - 3)(x + 2)(x + 3)$

So, the zeroes of  $x^3 + 2x^2 + kx - 18$  are 3, -2, -3.

### EXERCISE 2.4

- For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

(i)  $\frac{-8}{3}, \frac{4}{3}$

(ii)  $\frac{21}{8}, \frac{5}{16}$

(iii)  $-2\sqrt{3}, -9$

(iv)  $\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

- Given that the zeroes of the cubic polynomial  $x^3 - 6x^2 + 3x + 10$  are of the form  $a$ ,  $a + b$ ,  $a + 2b$  for some real numbers  $a$  and  $b$ , find the values of  $a$  and  $b$  as well as the zeroes of the given polynomial.

3. Given that  $\sqrt{2}$  is a zero of the cubic polynomial  $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$ , find its other two zeroes.
4. Find  $k$  so that  $x^2 + 2x + k$  is a factor of  $2x^4 + x^3 - 14x^2 + 5x + 6$ . Also find all the zeroes of the two polynomials.
5. Given that  $x - \sqrt{5}$  is a factor of the cubic polynomial  $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$ , find all the zeroes of the polynomial.
6. For which values of  $a$  and  $b$ , are the zeroes of  $q(x) = x^3 + 2x^2 + a$  also the zeroes of the polynomial  $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$ ? Which zeroes of  $p(x)$  are not the zeroes of  $q(x)$ ?

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## PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

### (A) Main Concepts and Results

- Two linear equations in the same two variables are said to form a pair of linear equations in two variables.

- The most general form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0,$$

where  $a_1, a_2, b_1, b_2, c_1, c_2$  are real numbers, such that  $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$ .

- A pair of linear equations is consistent if it has a solution – either a unique or infinitely many.

In case of infinitely many solutions, the pair of linear equations is also said to be dependent. Thus, in this case, the pair of linear equations is dependent and consistent.

- A pair of linear equations is inconsistent, if it has no solution.
- Let a pair of linear equations in two variables be  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$ .

(I) If  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ , then

- (i) the pair of linear equations is consistent,
- (ii) the graph will be a pair of lines intersecting at a unique point, which is the solution of the pair of equations.
- (II) If  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ , then
- (i) the pair of linear equations is inconsistent,
- (ii) the graph will be a pair of parallel lines and so the pair of equations will have no solution.
- (III) If  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , then
- (i) the pair of linear equations is dependent, and consistent,
- (ii) the graph will be a pair of coincident lines. Each point on the lines will be a solution, and so the pair of equations will have infinitely many solutions.
- A pair of linear equations can be solved algebraically by any of the following methods:
    - Substitution Method
    - Elimination Method
    - Cross- multiplication Method
  - The pair of linear equations can also be solved geometrically/graphically.

### (B) Multiple Choice Questions

Choose the correct answer from the given four options:

**Sample Question 1 :** The pair of equations  $5x - 15y = 8$  and  $3x - 9y = \frac{24}{5}$  has

- (A) one solution    (B) two solutions    (C) infinitely many solutions  
(D) no solution

**Solution :** Answer (C)



**Sample Question 2 :** The sum of the digits of a two-digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is

- (A) 25                      (B) 72                      (C) 63                      (D) 36

**Solution :** Answer (D)

### EXERCISE 3.1

Choose the correct answer from the given four options:

1. Graphically, the pair of equations

$$6x - 3y + 10 = 0$$

$$2x - y + 9 = 0$$

represents two lines which are

- (A) intersecting at exactly one point.                      (B) intersecting at exactly two points.  
 (C) coincident.                      (D) parallel.
2. The pair of equations  $x + 2y + 5 = 0$  and  $-3x - 6y + 1 = 0$  have  
 (A) a unique solution                      (B) exactly two solutions  
 (C) infinitely many solutions                      (D) no solution
3. If a pair of linear equations is consistent, then the lines will be  
 (A) parallel                      (B) always coincident  
 (C) intersecting or coincident                      (D) always intersecting
4. The pair of equations  $y = 0$  and  $y = -7$  has  
 (A) one solution                      (B) two solutions  
 (C) infinitely many solutions                      (D) no solution
5. The pair of equations  $x = a$  and  $y = b$  graphically represents lines which are  
 (A) parallel                      (B) intersecting at  $(b, a)$   
 (C) coincident                      (D) intersecting at  $(a, b)$
6. For what value of  $k$ , do the equations  $3x - y + 8 = 0$  and  $6x - ky = -16$  represent coincident lines?  
 (A)  $\frac{1}{2}$                       (B)  $-\frac{1}{2}$                       (C) 2                      (D) -2

7. If the lines given by  $3x + 2ky = 2$  and  $2x + 5y + 1 = 0$  are parallel, then the value of  $k$  is
- (A)  $\frac{-5}{4}$                       (B)  $\frac{2}{5}$                       (C)  $\frac{15}{4}$                       (D)  $\frac{3}{2}$
8. The value of  $c$  for which the pair of equations  $cx - y = 2$  and  $6x - 2y = 3$  will have infinitely many solutions is
- (A) 3                      (B) -3                      (C) -12                      (D) no value
9. One equation of a pair of dependent linear equations is  $-5x + 7y = 2$ . The second equation can be
- (A)  $10x + 14y + 4 = 0$                       (B)  $-10x - 14y + 4 = 0$   
(C)  $-10x + 14y + 4 = 0$                       (D)  $10x - 14y = -4$
10. A pair of linear equations which has a unique solution  $x = 2, y = -3$  is
- (A)  $x + y = -1$                       (B)  $2x + 5y = -11$   
 $2x - 3y = -5$                        $4x + 10y = -22$   
(C)  $2x - y = 1$                       (D)  $x - 4y - 14 = 0$   
 $3x + 2y = 0$                        $5x - y - 13 = 0$
11. If  $x = a, y = b$  is the solution of the equations  $x - y = 2$  and  $x + y = 4$ , then the values of  $a$  and  $b$  are, respectively
- (A) 3 and 5                      (B) 5 and 3  
(C) 3 and 1                      (D) -1 and -3
12. Aruna has only Re 1 and Rs 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs 75, then the number of Re 1 and Rs 2 coins are, respectively
- (A) 35 and 15                      (B) 35 and 20  
(C) 15 and 35                      (D) 25 and 25
13. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively
- (A) 4 and 24                      (B) 5 and 30  
(C) 6 and 36                      (D) 3 and 24

**(C) Short Answer Questions with Reasoning**

**Sample Question 1:** Is it true to say that the pair of equations

$$-x + 2y + 2 = 0 \text{ and } \frac{1}{2}x - \frac{1}{4}y - 1 = 0$$

has a unique solution? Justify your answer.

**Solution :** Yes.

$$\text{Here, } \frac{a_1}{a_2} = \frac{-1}{\frac{1}{2}} = -2, \quad \frac{b_1}{b_2} = \frac{2}{-\frac{1}{4}} = -8$$

As  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ , the pair of equations has a unique solution.

**Sample Question 2 :** Do the equations  $4x + 3y - 1 = 5$  and  $12x + 9y = 15$  represent a pair of coincident lines? Justify your answer.

**Solution :** No.

We may rewrite the equations as

$$4x + 3y = 6$$

$$12x + 9y = 15$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{1}{3} \text{ and } \frac{c_1}{c_2} = \frac{2}{5}$$

As  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ , the given equations do not represent a pair of coincident lines.

**Sample Question 3 :** Is the pair of equations  $x + 2y - 3 = 0$  and  $6y + 3x - 9 = 0$  consistent? Justify your answer.

**Solution :** Yes.

Rearranging the terms in the equations, we get

$$x + 2y - 3 = 0$$

$$3x + 6y - 9 = 0$$

Here,  $\frac{a_1}{a_2} = \frac{1}{3}$ ,  $\frac{b_1}{b_2} = \frac{1}{3}$ ,  $\frac{c_1}{c_2} = \frac{1}{3}$ . As  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , the pair of equations is consistent.

### EXERCISE 3.2

1. Do the following pair of linear equations have no solution? Justify your answer.

(i)  $2x + 4y = 3$

(ii)  $x = 2y$

$12y + 6x = 6$

$y = 2x$

(iii)  $3x + y - 3 = 0$

$2x + \frac{2}{3}y = 2$

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i)  $3x + \frac{1}{7}y = 3$

(ii)  $-2x - 3y = 1$

$7x + 3y = 7$

$6y + 4x = -2$

(iii)  $\frac{x}{2} + y + \frac{2}{5} = 0$

$4x + 8y + \frac{5}{16} = 0$

3. Are the following pair of linear equations consistent? Justify your answer.

(i)  $-3x - 4y = 12$

(ii)  $\frac{3}{5}x - y = \frac{1}{2}$

$4y + 3x = 12$

$\frac{1}{5}x - 3y = \frac{1}{6}$

(iii)  $2ax + by = a$

(iv)  $x + 3y = 11$

$4ax + 2by - 2a = 0$ ;  $a, b \neq 0$

$2(2x + 6y) = 22$

4. For the pair of equations

$\lambda x + 3y = -7$

$2x + 6y = 14$

to have infinitely many solutions, the value of  $\lambda$  should be 1. Is the statement true? Give reasons.

5. For all real values of  $c$ , the pair of equations

$$x - 2y = 8$$

$$5x - 10y = c$$

have a unique solution. Justify whether it is true or false.

6. The line represented by  $x = 7$  is parallel to the  $x$ -axis. Justify whether the statement is true or not.

### (D) Short Answer Questions

**Sample Question 1 :** For which values of  $p$  and  $q$ , will the following pair of linear equations have infinitely many solutions?

$$4x + 5y = 2$$

$$(2p + 7q)x + (p + 8q)y = 2q - p + 1.$$

**Solution :**

$$\text{Here, } \frac{a_1}{a_2} = \frac{4}{2p+7q}$$

$$\frac{b_1}{b_2} = \frac{5}{p+8q}$$

$$\frac{c_1}{c_2} = \frac{2}{2q-p+1}$$

For a pair of linear equations to have infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\text{So, } \frac{4}{2p+7q} = \frac{5}{p+8q} = \frac{2}{2q-p+1}$$

$$\text{So, } \frac{4}{2p+7q} = \frac{5}{p+8q} \text{ and } \frac{4}{2p+7q} = \frac{2}{2q-p+1}$$

$$\text{i.e., } 4p + 32q = 10p + 35q \text{ and } 8q - 4p + 4 = 4p + 14q$$

$$\text{i.e., } 6p + 3q = 0 \quad \text{and } 8p + 6q = 4$$

$$\text{i.e., } q = -2p \quad (1) \quad \text{and } 4p + 3q = 2 \quad (2)$$

Substituting the value of  $q$  obtained from Equation(1) in Equation(2), we get

$$4p - 6p = 2$$

$$\text{or } p = -1$$

Substituting the value of  $p$  in Equation (1), we get

$$q = 2$$

So, for  $p = -1, q = 2$ , the given pair of linear equations will have infinitely many solutions.

**Sample Question 2:** Solve the following pair of linear equations:

$$21x + 47y = 110$$

$$47x + 21y = 162$$

**Solution:** We have

$$21x + 47y = 110 \quad (1)$$

$$47x + 21y = 162 \quad (2)$$

Multiplying Equation (1) by 47 and Equation (2) by 21, we get

$$987x + 2209y = 5170 \quad (3)$$

$$987x + 441y = 3402 \quad (4)$$

Subtracting Equation (4) from Equation (3), we get

$$1768y = 1768$$

$$\text{or } y = 1$$

Substituting the value of  $y$  in Equation (1), we get

$$21x + 47 = 110$$

$$\text{or } 21x = 63$$

$$\text{or } x = 3$$

$$\text{So, } x = 3, y = 1$$

**Alternative Solution:** We have

$$21x + 47y = 110 \quad (1)$$

$$47x + 21y = 162 \quad (2)$$

Adding Equations (1) and (2), we have

$$68x + 68y = 272$$

or  $x + y = 4 \quad (5)$

Subtracting Equation (1) from Equation (2), we have

$$26x - 26y = 52$$

or  $x - y = 2 \quad (6)$

On adding and subtracting Equations (5) and (6), we get

$$x = 3, \quad y = 1$$

**Sample Question 3 :** Draw the graphs of the pair of linear equations  $x - y + 2 = 0$  and  $4x - y - 4 = 0$ . Calculate the area of the triangle formed by the lines so drawn and the  $x$ -axis.

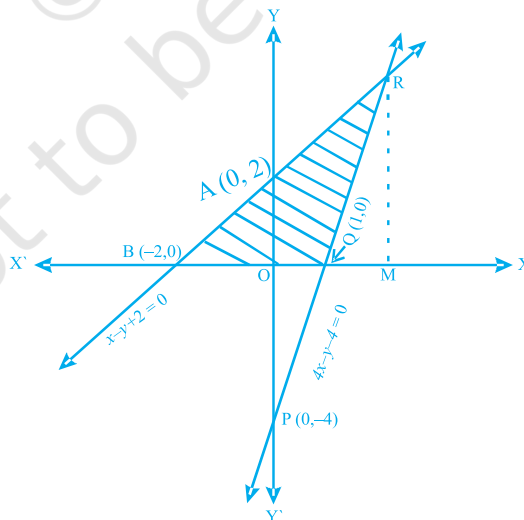
**Solution :**

For drawing the graphs of the given equations, we find two solutions of each of the equations, which are given in Table 3.1

**Table 3.1**

$x$	0	-2	$x$	0	1
$y = x + 2$	2	0	$y = 4x - 4$	-4	0

Plot the points A (0, 2), B (-2, 0), P (0, -4) and Q (1, 0) on the graph paper, and join the points to form the lines AB and PQ as shown in Fig 3.1



**Fig. 3.1**

We observe that there is a point R (2, 4) common to both the lines AB and PQ. The triangle formed by these lines and the  $x$ -axis is BQR.

The vertices of this triangle are B (-2, 0), Q (1, 0) and R (2, 4).

We know that;

$$\text{Area of triangle} = \frac{1}{2} \text{Base} \times \text{Altitude}$$

Here, Base = BQ = BO + OQ = 2 + 1 = 3 units.

Altitude = RM = Ordinate of R = 4 units.

$$\text{So, area of } \triangle \text{BQR} = \frac{1}{2} \times 3 \times 4 = 6 \text{ sq. units.}$$

### EXERCISE 3.3

1. For which value(s) of  $\lambda$ , do the pair of linear equations

$$\lambda x + y = \lambda^2 \text{ and } x + \lambda y = 1 \text{ have}$$

- (i) no solution?
  - (ii) infinitely many solutions?
  - (iii) a unique solution?
2. For which value(s) of  $k$  will the pair of equations
- $$kx + 3y = k - 3$$
- $$12x + ky = k$$
- have no solution?
3. For which values of  $a$  and  $b$ , will the following pair of linear equations have infinitely many solutions?
- $$x + 2y = 1$$
- $$(a - b)x + (a + b)y = a + b - 2$$
4. Find the value(s) of  $p$  in (i) to (iv) and  $p$  and  $q$  in (v) for the following pair of equations:
- (i)  $3x - y - 5 = 0$  and  $6x - 2y - p = 0$ ,  
if the lines represented by these equations are parallel.



- (ii)  $-x + py = 1$  and  $px - y = 1$ ,  
if the pair of equations has no solution.
- (iii)  $-3x + 5y = 7$  and  $2px - 3y = 1$ ,  
if the lines represented by these equations are intersecting at a unique point.
- (iv)  $2x + 3y - 5 = 0$  and  $px - 6y - 8 = 0$ ,  
if the pair of equations has a unique solution.
- (v)  $2x + 3y = 7$  and  $2px + py = 28 - qy$ ,  
if the pair of equations have infinitely many solutions.
5. Two straight paths are represented by the equations  $x - 3y = 2$  and  $-2x + 6y = 5$ . Check whether the paths cross each other or not.
6. Write a pair of linear equations which has the unique solution  $x = -1, y = 3$ . How many such pairs can you write?
7. If  $2x + y = 23$  and  $4x - y = 19$ , find the values of  $5y - 2x$  and  $\frac{y}{x} - 2$ .
8. Find the values of  $x$  and  $y$  in the following rectangle [see Fig. 3.2].

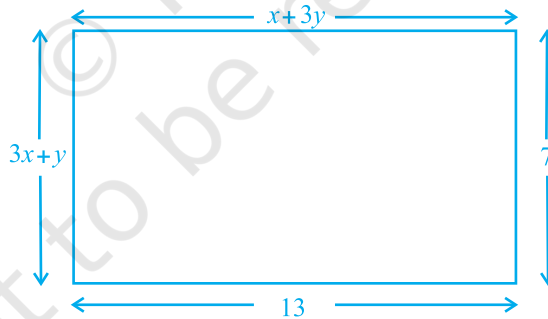


Fig. 3.2

9. Solve the following pairs of equations:

(i)  $x + y = 3.3$

$$\frac{0.6}{3x-2y} = -1, \quad 3x-2y \neq 0$$

(ii)  $\frac{x}{3} + \frac{y}{4} = 4$

$$\frac{5x}{6} - \frac{y}{8} = 4$$

(iii)  $4x + \frac{6}{y} = 15$

$6x - \frac{8}{y} = 14, y \neq 0$

(v)  $43x + 67y = -24$

$67x + 43y = 24$

(vii)  $\frac{2xy}{x+y} = \frac{3}{2}$

$\frac{xy}{2x-y} = \frac{-3}{10}, \quad x+y \neq 0, 2x-y \neq 0$

(iv)  $\frac{1}{2x} - \frac{1}{y} = -1$

$\frac{1}{x} + \frac{1}{2y} = 8, \quad x, y \neq 0$

(vi)  $\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2, \quad a, b \neq 0$

10. Find the solution of the pair of equations  $\frac{x}{10} + \frac{y}{5} - 1 = 0$  and  $\frac{x}{8} + \frac{y}{6} = 15$ .

Hence, find  $\lambda$ , if  $y = \lambda x + 5$ .

11. By the graphical method, find whether the following pair of equations are consistent or not. If consistent, solve them.

(i)  $3x + y + 4 = 0$

(ii)  $x - 2y = 6$

$6x - 2y + 4 = 0$

$3x - 6y = 0$

(iii)  $x + y = 3$

$3x + 3y = 9$

12. Draw the graph of the pair of equations  $2x + y = 4$  and  $2x - y = 4$ . Write the vertices of the triangle formed by these lines and the  $y$ -axis. Also find the area of this triangle.
13. Write an equation of a line passing through the point representing solution of the pair of linear equations  $x + y = 2$  and  $2x - y = 1$ . How many such lines can we find?
14. If  $x + 1$  is a factor of  $2x^3 + ax^2 + 2bx + 1$ , then find the values of  $a$  and  $b$  given that  $2a - 3b = 4$ .
15. The angles of a triangle are  $x$ ,  $y$  and  $40^\circ$ . The difference between the two angles  $x$  and  $y$  is  $30^\circ$ . Find  $x$  and  $y$ .

16. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?
17. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.
18. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.
19. There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.
20. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid Rs 22 for a book kept for six days, while Anand paid Rs 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.
21. In a competitive examination, one mark is awarded for each correct answer while  $\frac{1}{2}$  mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?
22. The angles of a cyclic quadrilateral ABCD are  
 $\angle A = (6x + 10)^\circ$ ,  $\angle B = (5x)^\circ$   
 $\angle C = (x + y)^\circ$ ,  $\angle D = (3y - 10)^\circ$   
Find  $x$  and  $y$ , and hence the values of the four angles.

### (E) Long Answer Questions

**Sample Question 1 :** Draw the graphs of the lines  $x = -2$  and  $y = 3$ . Write the vertices of the figure formed by these lines, the  $x$ -axis and the  $y$ -axis. Also, find the area of the figure.

#### Solution :

We know that the graph of  $x = -2$  is a line parallel to  $y$ -axis at a distance of 2 units to the left of it.

So, the line  $l$  is the graph of  $x = -2$  [see Fig. 3.3]

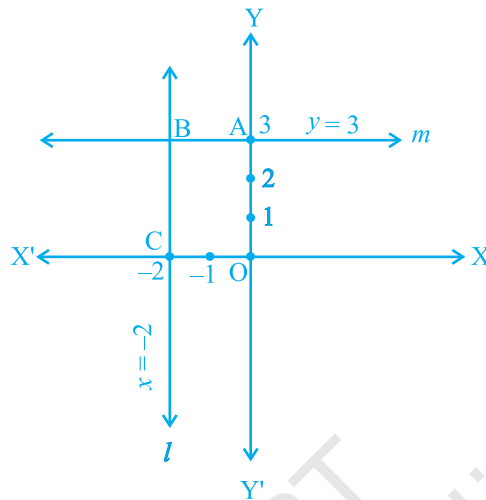


Fig. 3.3

The graph of  $y = 3$  is a line parallel to the  $x$ -axis at a distance of 3 units above it.

So, the line  $m$  is the graph of  $y = 3$ .

The figure enclosed by the lines  $x = -2$ ,  $y = 3$ , the  $x$ -axis and the  $y$ -axis is OABC, which is a rectangle. (Why?)

A is a point on the  $y$ -axis at a distance of 3 units above the  $x$ -axis. So, the coordinates of A are  $(0, 3)$ ;

C is a point on the  $x$ -axis at a distance of 2 units to the left of  $y$ -axis. So, the coordinates of C are  $(-2, 0)$

B is the solution of the pair of equations  $x = -2$  and  $y = 3$ . So, the coordinates of B are  $(-2, 3)$

So, the vertices of the rectangle OABC are O  $(0, 0)$ , A  $(0, 3)$ , B  $(-2, 3)$ , C  $(-2, 0)$

The length and breadth of this rectangle are 2 units and 3 units, respectively.

As the area of a rectangle = length  $\times$  breadth,

the area of rectangle OABC =  $2 \times 3 = 6$  sq. units.

**Sample Question 2:** Determine, algebraically, the vertices of the triangle formed by the lines

$$5x - y = 5, \quad x + 2y = 1 \text{ and } 6x + y = 17.$$

**Solution:**

The vertex of a triangle is the common solution of the two equations forming its two sides. So, solving the given equations pairwise will give the vertices of the triangle.

From the given equations, we will have the following three pairs of equations:

$$5x - y = 5 \quad \text{and} \quad x + 2y = 1$$

$$x + 2y = 1 \quad \text{and} \quad 6x + y = 17$$

$$5x - y = 5 \quad \text{and} \quad 6x + y = 17$$

Solving the pair of equations

$$5x - y = 5$$

$$x + 2y = 1$$

we get,  $x = 1, y = 0$

So, one vertex of the triangle is  $(1, 0)$

Solving the second pair of equations

$$x + 2y = 1$$

$$6x + y = 17$$

we get  $x = 3, y = -1$

So, another vertex of the triangle is  $(3, -1)$

Solving the third pair of equations

$$5x - y = 5$$

$$6x + y = 17,$$

we get  $x = 2, y = 5$ .

So, the third vertex of the triangle is  $(2, 5)$ . So, the three vertices of the triangle are  $(1, 0), (3, -1)$  and  $(2, 5)$ .

**Sample Question 3 :** Jamila sold a table and a chair for Rs 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got Rs 1065. Find the cost price of each.

**Solution :** Let the cost price of the table be Rs  $x$  and the cost price of the chair be Rs  $y$ .

The selling price of the table, when it is sold at a profit of 10%

$$= \text{Rs } x + \frac{10}{100}x = \text{Rs } \frac{110}{100}x$$

The selling price of the chair when it is sold at a profit of 25%

$$= \text{Rs } y + \frac{25}{100}y = \text{Rs } \frac{125}{100}y$$

$$\text{So, } \frac{110}{100}x + \frac{125}{100}y = 10500 \quad (1)$$

When the table is sold at a profit of 25%, its selling price = Rs  $\left(x + \frac{25}{100}x\right) = \text{Rs } \frac{125}{100}x$

When the chair is sold at a profit of 10%, its selling price = Rs  $\left(y + \frac{10}{100}y\right) = \text{Rs } \frac{110}{100}y$

$$\text{So, } \frac{125}{100}x + \frac{110}{100}y = 10650 \quad (2)$$

From Equations (1) and (2), we get

$$110x + 125y = 105000$$

$$\text{and } 125x + 110y = 106500$$

On adding and subtracting these equations, we get

$$235x + 235y = 211500$$

$$\text{and } 15x - 15y = 1500$$

$$\text{i.e., } x + y = 900 \quad (3)$$

$$\text{and } x - y = 100 \quad (4)$$

Solving Equations (3) and (4), we get

$$x = 500, y = 400$$

So, the cost price of the table is Rs 500 and the cost price of the chair is Rs 400.

**Sample Question 4:** It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. How long would it take for each pipe to fill the pool separately?

**Solution:**

Let the time taken by the pipe of larger diameter to fill the pool be  $x$  hours and that taken by the pipe of smaller diameter pipe alone be  $y$  hours.

In  $x$  hours, the pipe of larger diameter fills the pool.

So, in 1 hour the pipe of larger diameter fills  $\frac{1}{x}$  part of the pool, and so, in 4 hours, the pipe of larger diameter fills  $\frac{4}{x}$  parts of the pool.

Similarly, in 9 hours, the pipe of smaller diameter fills  $\frac{9}{y}$  parts of the pool.

According to the question,

$$\frac{4}{x} + \frac{9}{y} = \frac{1}{2} \quad (1)$$

Also, using both the pipes, the pool is filled in 12 hours.

$$\text{So, } \frac{12}{x} + \frac{12}{y} = 1 \quad (2)$$

Let  $\frac{1}{x} = u$  and  $\frac{1}{y} = v$ . Then Equations (1) and (2) become

$$4u + 9v = \frac{1}{2} \quad (3)$$

$$12u + 12v = 1 \quad (4)$$

Multiplying Equation (3) by 3 and subtracting Equation (4) from it, we get

$$15v = \frac{1}{2} \quad \text{or} \quad v = \frac{1}{30}$$

Substituting the value of  $v$  in Equation (4), we get  $u = \frac{1}{20}$

So,  $u = \frac{1}{20}, v = \frac{1}{30}$

So,  $\frac{1}{x} = \frac{1}{20}, \frac{1}{y} = \frac{1}{30}$

or,  $x = 20, y = 30.$

So, the pipe of larger diameter alone can fill the pool in 20 hours and the pipe of smaller diameter alone can fill the pool in 30 hours.

### EXERCISE 3.4

1. Graphically, solve the following pair of equations:

$$2x + y = 6$$

$$2x - y + 2 = 0$$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the  $x$ -axis and the lines with the  $y$ -axis.

2. Determine, graphically, the vertices of the triangle formed by the lines

$$y = x, \quad 3y = x, \quad x + y = 8$$

3. Draw the graphs of the equations  $x = 3, x = 5$  and  $2x - y - 4 = 0$ . Also find the area of the quadrilateral formed by the lines and the  $x$ -axis.
4. The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.
5. Determine, algebraically, the vertices of the triangle formed by the lines

$$3x - y = 3$$

$$2x - 3y = 2$$

$$x + 2y = 8$$

6. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw, and the remaining distance by bus.



On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

7. A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.
8. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.
9. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.
10. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs Rs 2530. Also, one reserved first class ticket and one reserved first class half ticket from A to B costs Rs 3810. Find the full first class fare from station A to B, and also the reservation charges for a ticket.
11. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum Rs 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs 1028. Find the cost price of the saree and the list price (price before discount) of the sweater.
12. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received Rs 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received Rs 20 more as annual interest. How much money did she invest in each scheme?
13. Vijay had some bananas, and he divided them into two lots A and B. He sold the first lot at the rate of Rs 2 for 3 bananas and the second lot at the rate of Re 1 per banana, and got a total of Rs 400. If he had sold the first lot at the rate of Re 1 per banana, and the second lot at the rate of Rs 4 for 5 bananas, his total collection would have been Rs 460. Find the total number of bananas he had.

## QUADRATIC EQUATIONS

### (A) Main Concepts and Results

- Quadratic equation : A quadratic equation in the variable  $x$  is of the form  $ax^2 + bx + c = 0$ , where  $a, b, c$  are real numbers and  $a \neq 0$ .
- Roots of a quadratic equation : A real number  $\alpha$  is said to be a root of the quadratic equation  $ax^2 + bx + c = 0$ , if  $a\alpha^2 + b\alpha + c = 0$ .
- The roots of the quadratic equation  $ax^2 + bx + c = 0$  are the same as the zeroes of the quadratic polynomial  $ax^2 + bx + c$ .
- Finding the roots of a quadratic equation by the method of factorisation : If we can factorise the quadratic polynomial  $ax^2 + bx + c$ , then the roots of the quadratic equation  $ax^2 + bx + c = 0$  can be found by equating to zero the linear factors of  $ax^2 + bx + c$ .
- Finding the roots of a quadratic equation by the method of completing the square : By adding and subtracting a suitable constant, we club the  $x^2$  and  $x$  terms in the quadratic equation so that they become a complete square, and solve for  $x$ .
- Quadratic Formula : If  $b^2 - 4ac \geq 0$ , then the real roots of the quadratic equation  $ax^2 + bx + c = 0$  are given by  $\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ .
- The expression  $b^2 - 4ac$  is called the discriminant of the quadratic equation.
- Existence of roots of a quadratic equation: A quadratic equation  $ax^2 + bx + c = 0$  has

- (i) two distinct real roots if  $b^2 - 4ac > 0$
- (ii) two equal real roots if  $b^2 - 4ac = 0$
- (iii) no real roots if  $b^2 - 4ac < 0$ .

### (B) Multiple Choice Questions

Choose the correct answer from the given four options:

**Sample Question 1 :** Which one of the following is not a quadratic equation?

- (A)  $(x + 2)^2 = 2(x + 3)$
- (B)  $x^2 + 3x = (-1)(1 - 3x)^2$
- (C)  $(x + 2)(x - 1) = x^2 - 2x - 3$
- (D)  $x^3 - x^2 + 2x + 1 = (x + 1)^3$

**Solution :** Answer (C)

**Sample Question 2 :** Which constant should be added and subtracted to solve the quadratic equation  $4x^2 - \sqrt{3}x - 5 = 0$  by the method of completing the square?

- (A)  $\frac{9}{16}$
- (B)  $\frac{3}{16}$
- (C)  $\frac{3}{4}$
- (D)  $\frac{\sqrt{3}}{4}$

**Solution :** Answer (B)

### EXERCISE 4.1

Choose the correct answer from the given four options in the following questions:

1. Which of the following is a quadratic equation?

- (A)  $x^2 + 2x + 1 = (4 - x)^2 + 3$
- (B)  $-2x^2 = (5 - x) \left( 2x - \frac{2}{5} \right)$
- (C)  $(k + 1)x^2 + \frac{3}{2}x = 7$ , where  $k = -1$
- (D)  $x^3 - x^2 = (x - 1)^3$

2. Which of the following is not a quadratic equation?

- (A)  $2(x - 1)^2 = 4x^2 - 2x + 1$
- (B)  $2x - x^2 = x^2 + 5$
- (C)  $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$
- (D)  $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

3. Which of the following equations has 2 as a root?

- (A)  $x^2 - 4x + 5 = 0$
- (B)  $x^2 + 3x - 12 = 0$
- (C)  $2x^2 - 7x + 6 = 0$
- (D)  $3x^2 - 6x - 2 = 0$

4. If  $\frac{1}{2}$  is a root of the equation  $x^2 + kx - \frac{5}{4} = 0$ , then the value of  $k$  is
- (A) 2                      (B) -2                      (C)  $\frac{1}{4}$                       (D)  $\frac{1}{2}$
5. Which of the following equations has the sum of its roots as 3?
- (A)  $2x^2 - 3x + 6 = 0$                       (B)  $-x^2 + 3x - 3 = 0$
- (C)  $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$                       (D)  $3x^2 - 3x + 3 = 0$
6. Values of  $k$  for which the quadratic equation  $2x^2 - kx + k = 0$  has equal roots is
- (A) 0 only                      (B) 4                      (C) 8 only                      (D) 0, 8
7. Which constant must be added and subtracted to solve the quadratic equation  $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$  by the method of completing the square?
- (A)  $\frac{1}{8}$                       (B)  $\frac{1}{64}$                       (C)  $\frac{1}{4}$                       (D)  $\frac{9}{64}$
8. The quadratic equation  $2x^2 - \sqrt{5}x + 1 = 0$  has
- (A) two distinct real roots                      (B) two equal real roots
- (C) no real roots                      (D) more than 2 real roots
9. Which of the following equations has two distinct real roots?
- (A)  $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$                       (B)  $x^2 + x - 5 = 0$
- (C)  $x^2 + 3x + 2\sqrt{2} = 0$                       (D)  $5x^2 - 3x + 1 = 0$
10. Which of the following equations has no real roots?
- (A)  $x^2 - 4x + 3\sqrt{2} = 0$                       (B)  $x^2 + 4x - 3\sqrt{2} = 0$
- (C)  $x^2 - 4x - 3\sqrt{2} = 0$                       (D)  $3x^2 + 4\sqrt{3}x + 4 = 0$

11.  $(x^2 + 1)^2 - x^2 = 0$  has

- (A) four real roots                      (B) two real roots  
 (C) no real roots                         (D) one real root.

### (C) Short Answer Questions with Reasoning

**Sample Question 1 :** Does  $(x - 1)^2 + 2(x + 1) = 0$  have a real root? Justify your answer.

**Solution :** No, since the equation is simplified to  $x^2 + 3 = 0$  whose discriminant is  $-12$ .

**Sample Question 2 :** Is the following statement 'True' or 'False'? Justify your answer. If in a quadratic equation the coefficient of  $x$  is zero, then the quadratic equation has no real roots.

**Solution :** False, since the discriminant in this case is  $-4ac$  which can still be non-negative if  $a$  and  $c$  are of opposite signs or if one of  $a$  or  $c$  is zero.

### EXERCISE 4.2

1. State whether the following quadratic equations have two distinct real roots. Justify your answer.

(i)  $x^2 - 3x + 4 = 0$

(ii)  $2x^2 + x - 1 = 0$

(iii)  $2x^2 - 6x + \frac{9}{2} = 0$

(iv)  $3x^2 - 4x + 1 = 0$

(v)  $(x + 4)^2 - 8x = 0$

(vi)  $(x - \sqrt{2})^2 - 2(x + 1) = 0$

(vii)  $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

(viii)  $x(1 - x) - 2 = 0$

(ix)  $(x - 1)(x + 2) + 2 = 0$

(x)  $(x + 1)(x - 2) + x = 0$

2. Write whether the following statements are true or false. Justify your answers.

- (i) Every quadratic equation has exactly one root.  
 (ii) Every quadratic equation has at least one real root.  
 (iii) Every quadratic equation has at least two roots.  
 (iv) Every quadratic equations has at most two roots.  
 (v) If the coefficient of  $x^2$  and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

- (vi) If the coefficient of  $x^2$  and the constant term have the same sign and if the coefficient of  $x$  term is zero, then the quadratic equation has no real roots.
3. A quadratic equation with integral coefficient has integral roots. Justify your answer.
  4. Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.
  5. Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?
  6. Is 0.2 a root of the equation  $x^2 - 0.4 = 0$ ? Justify.
  7. If  $b = 0, c < 0$ , is it true that the roots of  $x^2 + bx + c = 0$  are numerically equal and opposite in sign? Justify.

### (D) Short Answer Questions

**Sample Question 1 :** Find the roots of the quadratic equation  $2x^2 - \sqrt{5}x - 2 = 0$  using the quadratic formula.

**Solution :**  $b^2 - 4ac = 5 - 4 \times 2 \times (-2) = 21$

Therefore, the roots are  $\frac{\sqrt{5} \pm \sqrt{21}}{4}$ , i.e.,  $\frac{\sqrt{5} + \sqrt{21}}{4}$  and  $\frac{\sqrt{5} - \sqrt{21}}{4}$

**Sample Question 2 :** Find the roots of  $6x^2 - \sqrt{2}x - 2 = 0$  by the factorisation of the corresponding quadratic polynomial.

**Solution :**

$$\begin{aligned} 6x^2 - \sqrt{2}x - 2 &= 6x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 2 \\ &= 3x(2x - \sqrt{2}) + \sqrt{2}(2x - \sqrt{2}) \\ &= (3x + \sqrt{2})(2x - \sqrt{2}) \end{aligned}$$

Now,  $6x^2 - \sqrt{2}x - 2 = 0$  gives  $(3x + \sqrt{2})(2x - \sqrt{2}) = 0$ , i.e.,  $3x + \sqrt{2} = 0$  or  $2x - \sqrt{2} = 0$

So, the roots are  $-\frac{\sqrt{2}}{3}$  and  $\frac{\sqrt{2}}{2}$ .

### EXERCISE 4.3

1. Find the roots of the quadratic equations by using the quadratic formula in each of the following:

(i)  $2x^2 - 3x - 5 = 0$

(ii)  $5x^2 + 13x + 8 = 0$

(iii)  $-3x^2 + 5x + 12 = 0$

(iv)  $-x^2 + 7x - 10 = 0$

(v)  $x^2 + 2\sqrt{2}x - 6 = 0$

(vi)  $x^2 - 3\sqrt{5}x + 10 = 0$

(vii)  $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

2. Find the roots of the following quadratic equations by the factorisation method:

(i)  $2x^2 + \frac{5}{3}x - 2 = 0$

(ii)  $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

(iii)  $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

(iv)  $3x^2 + 5\sqrt{5}x - 10 = 0$

(v)  $21x^2 - 2x + \frac{1}{21} = 0$

### (E) Long Answer Questions

**Sample Question 1 :** Check whether the equation  $6x^2 - 7x + 2 = 0$  has real roots, and if it has, find them by the method of completing the squares.

**Solution :** The discriminant  $= b^2 - 4ac = 49 - 4 \times 6 \times 2 = 1 > 0$

So, the given equation has two distinct real roots.

Now,  $6x^2 - 7x + 2 = 0$

i.e.,  $36x^2 - 42x + 12 = 0$

i.e.,  $6x - \frac{7}{2} + 12 - \frac{49}{4} = 0$

i.e.,  $6x - \frac{7}{2} - \frac{1}{2} = 0$  or  $\left(6x - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

The roots are given by  $6x - \frac{7}{2} = \pm \frac{1}{2}$

i.e.,  $6x = 4, 3$

i.e.,  $x = \frac{2}{3}, \frac{1}{2}$ .

**Sample Question 2 :** Had Ajita scored 10 more marks in her mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?

**Solution :** Let her actual marks be  $x$

Therefore,  $9(x+10) = x^2$

i.e.,  $x^2 - 9x - 90 = 0$

i.e.,  $x^2 - 15x + 6x - 90 = 0$

i.e.,  $x(x-15) + 6(x-15) = 0$

i.e.,  $(x+6)(x-15) = 0$

Therefore,  $x = -6$  or  $x = 15$

Since  $x$  is the marks obtained,  $x \neq -6$ . Therefore,  $x = 15$ .

So, Ajita got 15 marks in her mathematics test.

**Sample Question 3 :** A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is its original average speed?

**Solution :** Let its original average speed be  $x$  km/h. Therefore,

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

i.e.,  $\frac{7}{x} + \frac{8}{x+6} = \frac{3}{9} = \frac{1}{3}$

i.e.,  $\frac{7(x+6)+8x}{x(x+6)} = \frac{1}{3}$



$$\text{i.e.,} \quad 21(x + 6) + 24x = x(x + 6)$$

$$\text{i.e.,} \quad 21x + 126 + 24x = x^2 + 6x$$

$$\text{i.e.,} \quad x^2 - 39x - 126 = 0$$

$$\text{i.e.,} \quad (x + 3)(x - 42) = 0$$

$$\text{i.e.,} \quad x = -3 \text{ or } x = 42$$

Since  $x$  is the average speed of the train,  $x$  cannot be negative.

Therefore,  $x = 42$ .

So, the original average speed of the train is 42 km/h.

### EXERCISE 4.4

1. Find whether the following equations have real roots. If real roots exist, find them.

$$(i) \quad 8x^2 + 2x - 3 = 0$$

$$(ii) \quad -2x^2 + 3x + 2 = 0$$

$$(iii) \quad 5x^2 - 2x - 10 = 0$$

$$(iv) \quad \frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5$$

$$(v) \quad x^2 + 5\sqrt{5}x - 70 = 0$$

2. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.
3. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.
4. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.
5. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?
6. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.

7. In the centre of a rectangular lawn of dimensions  $50\text{ m} \times 40\text{ m}$ , a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be  $1184\text{ m}^2$  [see Fig. 4.1]. Find the length and breadth of the pond.

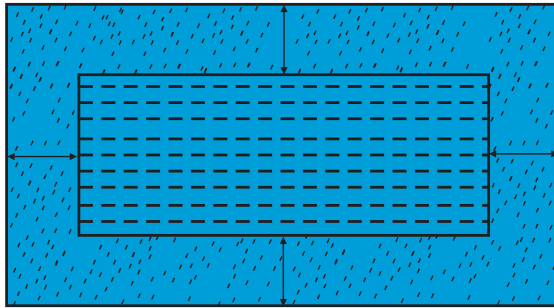


Fig. 4.1

8. At  $t$  minutes past 2 pm, the time needed by the minutes hand of a clock to show 3 pm was found to be 3 minutes less than  $\frac{t^2}{4}$  minutes. Find  $t$ .

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## ARITHMETIC PROGRESSIONS

### (A) Main Concepts and Results

- An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number  $d$  to the preceding term, except the first term  $a$ . The fixed number  $d$  is called its common difference.

The general form of an AP is  $a, a + d, a + 2d, a + 3d, \dots$

- In the list of numbers  $a_1, a_2, a_3, \dots$  if the differences  $a_2 - a_1, a_3 - a_2, a_4 - a_3, \dots$  give the same value, i.e., if  $a_{k+1} - a_k$  is the same for different values of  $k$ , then the given list of numbers is an AP.
- The  $n^{\text{th}}$  term  $a_n$  (or the general term) of an AP is  $a_n = a + (n - 1) d$ , where  $a$  is the first term and  $d$  is the common difference. Note that  $a_1 = a$ .
- The sum  $S_n$  of the first  $n$  terms of an AP is given by

$$S_n = \frac{n}{2} [2a + (n - 1) d]$$

If  $l$  is the last term of an AP of  $n$  terms, then the sum of all the terms can also be given by

$$S_n = \frac{n}{2} [a + l]$$

Sometimes  $S_n$  is also denoted by  $S$ .

- If  $S_n$  is the sum of the first  $n$  terms of an AP, then its  $n^{\text{th}}$  term  $a_n$  is given by

$$a_n = S_n - S_{n-1}$$

### (B) Multiple Choice Questions

Choose the correct answer from the given four options:

**Sample Question 1 :** The 10<sup>th</sup> term of the AP: 5, 8, 11, 14, ... is

- (A) 32            (B) 35            (C) 38            (D) 185

**Solution :** Answer (A)

**Sample Question 2 :** In an AP if  $a = -7.2$ ,  $d = 3.6$ ,  $a_n = 7.2$ , then  $n$  is

- (A) 1            (B) 3            (C) 4            (D) 5

**Solution :** Answer (D)

### EXERCISE 5.1

Choose the correct answer from the given four options:

- In an AP, if  $d = -4$ ,  $n = 7$ ,  $a_n = 4$ , then  $a$  is  
 (A) 6            (B) 7            (C) 20            (D) 28
- In an AP, if  $a = 3.5$ ,  $d = 0$ ,  $n = 101$ , then  $a_n$  will be  
 (A) 0            (B) 3.5            (C) 103.5            (D) 104.5
- The list of numbers  $-10, -6, -2, 2, \dots$  is  
 (A) an AP with  $d = -16$   
 (B) an AP with  $d = 4$   
 (C) an AP with  $d = -4$   
 (D) not an AP
- The 11<sup>th</sup> term of the AP:  $-5, \frac{-5}{2}, 0, \frac{5}{2}, \dots$  is  
 (A)  $-20$             (B)  $20$             (C)  $-30$             (D)  $30$
- The first four terms of an AP, whose first term is  $-2$  and the common difference is  $-2$ , are

- (A)  $-2, 0, 2, 4$   
(B)  $-2, 4, -8, 16$   
(C)  $-2, -4, -6, -8$   
(D)  $-2, -4, -8, -16$
6. The 21<sup>st</sup> term of the AP whose first two terms are  $-3$  and  $4$  is  
(A) 17            (B) 137            (C) 143            (D)  $-143$
7. If the 2<sup>nd</sup> term of an AP is 13 and the 5<sup>th</sup> term is 25, what is its 7<sup>th</sup> term?  
(A) 30            (B) 33            (C) 37            (D) 38
8. Which term of the AP: 21, 42, 63, 84,... is 210?  
(A) 9<sup>th</sup>            (B) 10<sup>th</sup>            (C) 11<sup>th</sup>            (D) 12<sup>th</sup>
9. If the common difference of an AP is 5, then what is  $a_{18} - a_{13}$ ?  
(A) 5            (B) 20            (C) 25            (D) 30
10. What is the common difference of an AP in which  $a_{18} - a_{14} = 32$ ?  
(A) 8            (B)  $-8$             (C)  $-4$             (D) 4
11. Two APs have the same common difference. The first term of one of these is  $-1$  and that of the other is  $-8$ . Then the difference between their 4<sup>th</sup> terms is  
(A)  $-1$             (B)  $-8$             (C) 7            (D)  $-9$
12. If 7 times the 7<sup>th</sup> term of an AP is equal to 11 times its 11<sup>th</sup> term, then its 18<sup>th</sup> term will be  
(A) 7            (B) 11            (C) 18            (D) 0
13. The 4<sup>th</sup> term from the end of the AP:  $-11, -8, -5, \dots, 49$  is  
(A) 37            (B) 40            (C) 43            (D) 58
14. The famous mathematician associated with finding the sum of the first 100 natural numbers is  
(A) Pythagoras            (B) Newton  
(C) Gauss            (D) Euclid
15. If the first term of an AP is  $-5$  and the common difference is 2, then the sum of the first 6 terms is  
(A) 0            (B) 5            (C) 6            (D) 15

- 16.** The sum of first 16 terms of the AP: 10, 6, 2, ... is  
 (A) -320            (B) 320            (C) -352            (D) -400
- 17.** In an AP if  $a = 1$ ,  $a_n = 20$  and  $S_n = 399$ , then  $n$  is  
 (A) 19            (B) 21            (C) 38            (D) 42
- 18.** The sum of first five multiples of 3 is  
 (A) 45            (B) 55            (C) 65            (D) 75

### (C) Short Answer Questions with Reasoning

**Sample Question 1:** In the AP: 10, 5, 0, -5, ... the common difference  $d$  is equal to 5.

Justify whether the above statement is true or false.

**Solution :**

$$a_2 - a_1 = 5 - 10 = -5$$

$$a_3 - a_2 = 0 - 5 = -5$$

$$a_4 - a_3 = -5 - 0 = -5$$

Although the given list of numbers forms an AP, it is with  $d = -5$  and not with  $d = 5$

So, the given statement is false.

**Sample Question 2 :** Divya deposited Rs 1000 at compound interest at the rate of 10% per annum. The amounts at the end of first year, second year, third year, ..., form an AP. Justify your answer.

**Solution :** Amount at the end of the 1st year = Rs 1100

Amount at the end of the 2nd year = Rs 1210

Amount at the end of 3rd year = Rs 1331 and so on.

So, the amount (in Rs) at the end of 1st year, 2nd year, 3rd year, ... are

1100, 1210, 1331, ...

Here,  $a_2 - a_1 = 110$

$a_3 - a_2 = 121$

As,  $a_2 - a_1 \neq a_3 - a_2$ , it does not form an AP.

**Sample Question 3:** The  $n^{\text{th}}$  term of an AP cannot be  $n^2 + 1$ . Justify your answer.

**Solution :**

$$\text{Here, } a_n = n^2 + 1$$

$$\text{So, } a_1 = 1^2 + 1 = 2$$

$$a_2 = 2^2 + 1 = 5$$

$$a_3 = 3^2 + 1 = 10$$

$$\vdots$$

List of numbers becomes 2, 5, 10, ...

Here,  $5 - 2 \neq 10 - 5$ , so it does not form an AP.

**Alternative Solution 1:**

We know that in an AP,  $d = a_n - a_{n-1}$

$$\text{Here, } a_n = n^2 + 1$$

$$\begin{aligned} \text{So, } a_n - a_{n-1} &= (n^2 + 1) - (n-1)^2 + 1 \\ &= 2n - 1 \end{aligned}$$

As  $a_n - a_{n-1}$  depends upon  $n$ ,  $d$  is not a fixed number.

So,  $a_n = n^2 + 1$  cannot be the  $n^{\text{th}}$  term of an AP.

**Alternative Solution 2:**

We know that in an AP

$a_n = a + (n-1)d$ . We observe that  $a_n$  is a linear polynomial in  $n$ .

Here,  $a_n = n^2 + 1$  is not a linear polynomial in  $n$ . So, it cannot be the  $n^{\text{th}}$  term of an AP.

## EXERCISE 5.2

1. Which of the following form an AP? Justify your answer.
  - (i)  $-1, -1, -1, -1, \dots$
  - (ii)  $0, 2, 0, 2, \dots$
  - (iii)  $1, 1, 2, 2, 3, 3, \dots$
  - (iv)  $11, 22, 33, \dots$
  - (v)  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$
  - (vi)  $2, 2^2, 2^3, 2^4, \dots$
  - (vii)  $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \dots$
  
2. Justify whether it is true to say that  $-1, -\frac{3}{2}, -2, -\frac{5}{2}, \dots$  forms an AP as  $a_2 - a_1 = a_3 - a_2$ .
  
3. For the AP:  $-3, -7, -11, \dots$ , can we find directly  $a_{30} - a_{20}$  without actually finding  $a_{30}$  and  $a_{20}$ ? Give reasons for your answer.
  
4. Two APs have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10<sup>th</sup> terms is the same as the difference between their 21<sup>st</sup> terms, which is the same as the difference between any two corresponding terms. Why?
  
5. Is 0 a term of the AP:  $31, 28, 25, \dots$ ? Justify your answer.
  
6. The taxi fare after each km, when the fare is Rs 15 for the first km and Rs 8 for each additional km, does not form an AP as the total fare (in Rs) after each km is
 
$$15, 8, 8, 8, \dots$$
 Is the statement true? Give reasons.
  
7. In which of the following situations, do the lists of numbers involved form an AP? Give reasons for your answers.
  - (i) The fee charged from a student every month by a school for the whole session, when the monthly fee is Rs 400.



- (ii) The fee charged every month by a school from Classes I to XII, when the monthly fee for Class I is Rs 250, and it increases by Rs 50 for the next higher class.
- (iii) The amount of money in the account of Varun at the end of every year when Rs 1000 is deposited at simple interest of 10% per annum.
- (iv) The number of bacteria in a certain food item after each second, when they double in every second.
8. Justify whether it is true to say that the following are the  $n^{\text{th}}$  terms of an AP.
- (i)  $2n-3$                       (ii)  $3n^2+5$                       (iii)  $1+n+n^2$

#### (D) Short Answer Questions

**Sample Question 1 :** If the numbers  $n - 2$ ,  $4n - 1$  and  $5n + 2$  are in AP, find the value of  $n$ .

**Solution :**

As  $n - 2$ ,  $4n - 1$ ,  $5n + 2$  are in AP,

$$\text{so } (4n - 1) - (n - 2) = (5n + 2) - (4n - 1)$$

$$\text{i.e., } 3n + 1 = n + 3$$

$$\text{i.e., } n = 1$$

**Sample Question 2 :** Find the value of the middle most term (s) of the AP :  $-11, -7, -3, \dots, 49$ .

**Solution :**

$$\text{Here, } a = -11, \quad d = -7 - (-11) = 4, \quad a_n = 49$$

$$\text{We have } a_n = a + (n - 1) d$$

$$\text{So, } 49 = -11 + (n - 1) \times 4$$

$$\text{i.e., } 60 = (n - 1) \times 4$$

$$\text{i.e., } n = 16$$

As  $n$  is an even number, there will be two middle terms which are

$\frac{16}{2}$ th and  $\left(\frac{16}{2} + 1\right)$ th, i.e., the 8<sup>th</sup> term and the 9<sup>th</sup> term.

$$a_8 = a + 7d = -11 + 7 \times 4 = 17$$

$$a_9 = a + 8d = -11 + 8 \times 4 = 21$$

So, the values of the two middle most terms are 17 and 21, respectively.

**Sample Question 3:** The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.

**Solution :** Let the three terms in AP be

$$a - d, a, a + d.$$

So,  $a - d + a + a + d = 33$

or  $a = 11$

Also,  $(a - d)(a + d) = a + 29$

i.e.,  $a^2 - d^2 = a + 29$

i.e.,  $121 - d^2 = 11 + 29$

i.e.,  $d^2 = 81$

i.e.,  $d = \pm 9$

So there will be two APs and they are : 2, 11, 20, ...

and 20, 11, 2, ...

### EXERCISE 5.3

1. Match the APs given in column A with suitable common differences given in column B.

**Column A**

(A<sub>1</sub>) 2, -2, -6, -10, ...

(A<sub>2</sub>)  $a = -18, n = 10, a_n = 0$

(A<sub>3</sub>)  $a = 0, a_{10} = 6$

(A<sub>4</sub>)  $a_2 = 13, a_4 = 3$

**Column B**

(B<sub>1</sub>)  $\frac{2}{3}$

(B<sub>2</sub>) -5

(B<sub>3</sub>) 4

(B<sub>4</sub>) -4

(B<sub>5</sub>) 2

(B<sub>6</sub>)  $\frac{1}{2}$

(B<sub>7</sub>) 5

2. Verify that each of the following is an AP, and then write its next three terms.

(i)  $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \dots$

(ii)  $5, \frac{14}{3}, \frac{13}{3}, 4, \dots$

(iii)  $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \dots$

(iv)  $a + b, (a + 1) + b, (a + 1) + (b + 1), \dots$

(v)  $a, 2a + 1, 3a + 2, 4a + 3, \dots$

3. Write the first three terms of the APs when  $a$  and  $d$  are as given below:

(i)  $a = \frac{1}{2}, d = -\frac{1}{6}$

(ii)  $a = -5, d = -3$

(iii)  $a = \sqrt{2}, d = \frac{1}{\sqrt{2}}$

4. Find  $a, b$  and  $c$  such that the following numbers are in AP:  $a, 7, b, 23, c$ .

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

6. The 26<sup>th</sup>, 11<sup>th</sup> and the last term of an AP are 0, 3 and  $-\frac{1}{5}$ , respectively. Find the common difference and the number of terms.

7. The sum of the 5<sup>th</sup> and the 7<sup>th</sup> terms of an AP is 52 and the 10<sup>th</sup> term is 46. Find the AP.

8. Find the 20<sup>th</sup> term of the AP whose 7<sup>th</sup> term is 24 less than the 11<sup>th</sup> term, first term being 12.

9. If the 9<sup>th</sup> term of an AP is zero, prove that its 29<sup>th</sup> term is twice its 19<sup>th</sup> term.

10. Find whether 55 is a term of the AP: 7, 10, 13, --- or not. If yes, find which term it is.

11. Determine  $k$  so that  $k^2 + 4k + 8$ ,  $2k^2 + 3k + 6$ ,  $3k^2 + 4k + 4$  are three consecutive terms of an AP.
12. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.
13. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.
14. If the  $n$ th terms of the two APs: 9, 7, 5, ... and 24, 21, 18, ... are the same, find the value of  $n$ . Also find that term.
15. If sum of the 3<sup>rd</sup> and the 8<sup>th</sup> terms of an AP is 7 and the sum of the 7<sup>th</sup> and the 14<sup>th</sup> terms is  $-3$ , find the 10<sup>th</sup> term.
16. Find the 12<sup>th</sup> term from the end of the AP:  $-2, -4, -6, \dots, -100$ .
17. Which term of the AP: 53, 48, 43, ... is the first negative term?
18. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?
19. Find the sum of the two middle most terms of the AP:  $-\frac{4}{3}, -1, -\frac{2}{3}, \dots, 4\frac{1}{3}$ .
20. The first term of an AP is  $-5$  and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.
21. Find the sum:
- $1 + (-2) + (-5) + (-8) + \dots + (-236)$
  - $4 - \frac{1}{n} + 4 - \frac{2}{n} + 4 - \frac{3}{n} + \dots$  upto  $n$  terms
  - $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \dots$  to 11 terms.
22. Which term of the AP:  $-2, -7, -12, \dots$  will be  $-77$ ? Find the sum of this AP upto the term  $-77$ .
23. If  $a_n = 3 - 4n$ , show that  $a_1, a_2, a_3, \dots$  form an AP. Also find  $S_{20}$ .
24. In an AP, if  $S_n = n(4n + 1)$ , find the AP.

25. In an AP, if  $S_n = 3n^2 + 5n$  and  $a_k = 164$ , find the value of  $k$ .
26. If  $S_n$  denotes the sum of first  $n$  terms of an AP, prove that
- $$S_{12} = 3(S_8 - S_4)$$
27. Find the sum of first 17 terms of an AP whose 4<sup>th</sup> and 9<sup>th</sup> terms are  $-15$  and  $-30$  respectively.
28. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.
29. Find the sum of all the 11 terms of an AP whose middle most term is 30.
30. Find the sum of last ten terms of the AP: 8, 10, 12, ---, 126.
31. Find the sum of first seven numbers which are multiples of 2 as well as of 9.
- [Hint:** Take the LCM of 2 and 9]
32. How many terms of the AP:  $-15, -13, -11, \dots$  are needed to make the sum  $-55$ ? Explain the reason for double answer.
33. The sum of the first  $n$  terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first  $2n$  terms of another AP whose first term is  $-30$  and the common difference is 8. Find  $n$ .
34. Kanika was given her pocket money on Jan 1<sup>st</sup>, 2008. She puts Re 1 on Day 1, Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end of the month she still had Rs 100 with her. How much was her pocket money for the month?
35. Yasmeen saves Rs 32 during the first month, Rs 36 in the second month and Rs 40 in the third month. If she continues to save in this manner, in how many months will she save Rs 2000?

### (E) Long Answer Questions

**Sample Question 1:** The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last terms to the product of the two middle terms is 7 : 15. Find the numbers.

**Solution:** Let the four consecutive numbers in AP be

$$a - 3d, a - d, a + d, a + 3d.$$

$$\text{So, } a - 3d + a - d + a + d + a + 3d = 32$$

$$\text{or } 4a = 32$$

$$\text{or } a = 8$$

$$\text{Also, } \frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\text{or, } \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$$

$$\text{or, } 15a^2 - 135d^2 = 7a^2 - 7d^2$$

$$\text{or, } 8a^2 - 128d^2 = 0$$

$$\text{or, } d^2 = \frac{8 \times 8 \times 8}{128} = 4$$

$$\text{or, } d = \pm 2$$

So, when  $a = 8$ ,  $d = 2$ , the numbers are 2, 6, 10, 14.

**Sample Question 2:** Solve the equation :

$$1 + 4 + 7 + 10 + \dots + x = 287$$

**Solution :**

Here, 1, 4, 7, 10, ...,  $x$  form an AP with  $a = 1$ ,  $d = 3$ ,  $a_n = x$

We have,  $a_n = a + (n - 1)d$

$$\text{So, } x = 1 + (n - 1) \times 3 = 3n - 2$$

$$\text{Also, } S = \frac{n}{2}(a + l)$$

$$\text{So, } 287 = \frac{n}{2}(1 + x)$$

$$= \frac{n}{2}(1+3n-2)$$

$$\text{or, } 574 = n(3n-1)$$

$$\text{or, } 3n^2 - n - 574 = 0$$

$$\text{Therefore, } n = \frac{1 \pm \sqrt{1+6888}}{6}$$

$$= \frac{1 \pm 83}{6} = \frac{84}{6}, \frac{-82}{6}$$

$$= 14, \frac{-41}{3}$$

As  $n$  cannot be negative, so  $n = 14$

Therefore,  $x = 3n - 2 = 3 \times 14 - 2 = 40$ .

**Alternative solution:**

Here, 1, 4, 7, 10, ...  $x$  form an AP with  $a = 1$ ,  $d = 3$ ,  $S = 287$

$$\text{We have, } S = \frac{n}{2} [2a + (n-1)d]$$

$$\text{So, } 287 = \frac{n}{2} [2 + (n-1) \times 3]$$

$$\text{or, } 574 = n(3n-1)$$

$$\text{or, } 3n^2 - n - 574 = 0$$

Now proceed as above.

### EXERCISE 5.4

1. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

## 2. Find the

- (i) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.
- (ii) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .
- (iii) sum of those integers from 1 to 500 which are multiples of 2 or 5.

**[Hint (iii) :** These numbers will be : multiples of 2 + multiples of 5 – multiples of 2 as well as of 5 ]

- 3. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15<sup>th</sup> term.
- 4. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

## 5. Find the sum of the integers between 100 and 200 that are

- (i) divisible by 9
- (ii) not divisible by 9

**[Hint (ii) :** These numbers will be : Total numbers – Total numbers divisible by 9]

- 6. The ratio of the 11<sup>th</sup> term to the 18<sup>th</sup> term of an AP is 2 : 3. Find the ratio of the 5<sup>th</sup> term to the 21<sup>st</sup> term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.
- 7. Show that the sum of an AP whose first term is  $a$ , the second term  $b$  and the last term  $c$ , is equal to

$$\frac{(a+c)(b+c-2a)}{2(b-a)}$$

## 8. Solve the equation

$$-4 + (-1) + 2 + \dots + x = 437$$

- 9. Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, what amount will be paid by him in the 30<sup>th</sup> instalment? What amount of loan does he still have to pay after the 30<sup>th</sup> instalment?



- 10.** The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

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Following hand written questions are a must for class 9th and 10 students for Summer vacation HW. Rest are class wise questions where the students have to solve as many questions of their ability. (All to be done in usual HW copy):

1)  $(a+b)^2 - (a-b)^2 = \dots\dots\dots$

2)  $(a+b)^2 - 4ab = \dots\dots\dots$

3)  $(x + \frac{1}{x})^2 = \dots\dots\dots$

4)  $(x^2 - y^2) \div (x - y) = \dots\dots\dots$

5) Factorize  $x^3 + x^2 + x$ .

6) Factorize  $3x^2 - 243$ .

7) Factorize  $x^2 + 5x + 6$

8) Factorize  $x^2 - x - 6$

9) Solve:  $3x - 8 = x + 6$

10) Find y if  $3y - 6 = 0$

11) What is the degree of zero polynomial?

12) Find the degree of the polynomial  $x^3 - 3x + 6$ .

13) Locate the point on the x-axis for  $\sqrt{3}$ .

14) Locate the points  $(-3, 2)$ ,  $(3, -2)$ ,  $(-3, -2)$  and  $(3, 2)$  on the co-ordinate plane.

15)  $3x + 2y = 8$ . Find y if  $x = 2$ .

16) If  $a + b = 9$  and  $a^2 + b^2 = 39$ , find ab.

17) Find the LCM of  $x^2 - y^2$  and  $x + y$ .

18) Simplify:  $(x^2 - y^2) \times (\frac{9x + 9y}{3x - 3y})$

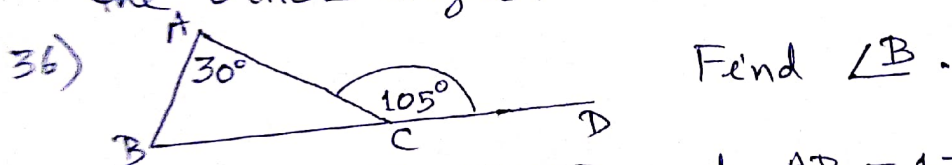
19) Find  $\frac{1}{x-y} + \frac{1}{x+y}$

20) Solve:  $\frac{3x}{3x+1} = \frac{x-1}{x+1}$

21) Prime factorize 540.

22) Find HCF of 39 and 17.

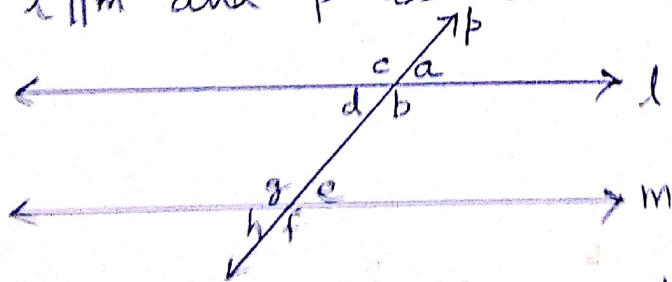
- 23) Find the HCF of 54 and 150.
- 24) Find the LCM of 17 and 19.
- 25) Find the LCM of 24 and 108.
- 26) Express  $\frac{1}{3}$  into decimal number.
- 27) Change  $\frac{1}{5}$  into percent.
- 28) Change 0.75 into rational number in  $\frac{p}{q}$  form.
- 29) Multiply :  $(-6)^3 \times \left(\frac{1}{-3}\right)^4 \times \frac{3}{8}$ .
- 30) Multiply  $\sqrt{8} \times \sqrt{18}$
- 31) Rationalize :  $\frac{1}{3-2\sqrt{2}}$
- 32) How many lines can pass through a point in a plane?
- 33) How many lines can pass through two points in a plane?
- 34) What is the sum of the three angles (interior) of a triangle?
- 35) In a right triangle, if one angle is  $46^\circ$ , what are the other angles.



- 37) If  $\triangle ABC \cong \triangle DEF$  and  $AB = 13 \text{ cm}$ ,  $\angle B = 22^\circ$ , find  $DE$  and  $\angle E$ .

- 38) The angles of a triangle are in ratio 3:4:5. Find the angles.

- 39)  $l \parallel m$  and  $p$  is transversal

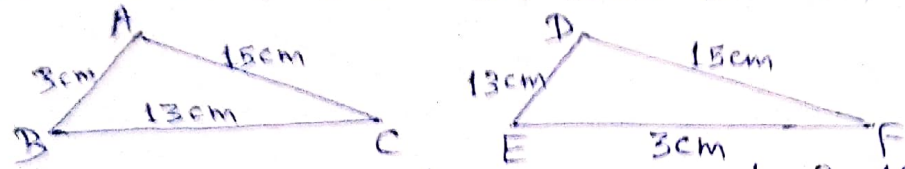


- Write the pairs of corresponding angles.
- Write the pairs of alternate exterior angles.
- Find  $e + b$ .
- Write all angles equal to  $f$ .

40) If  $(x, 12, 13)$  is a Pythagorean triplets, find  $x$ .

41) In  $\triangle DEF$ ,  $DE = DF$  and  $\angle E = 57^\circ$ . Find  $\angle F$ .

42)

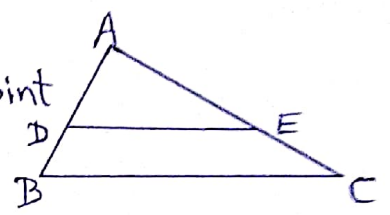


i) Are the triangles congruent? If yes, write their properties.

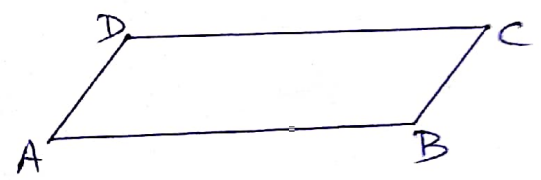
ii) Which one will be true:

- a)  $\triangle ABC \cong \triangle FDE$ ,
- b)  $\triangle ABC \cong \triangle DFE$ ,
- c)  $\triangle CAB \cong \triangle EFD$

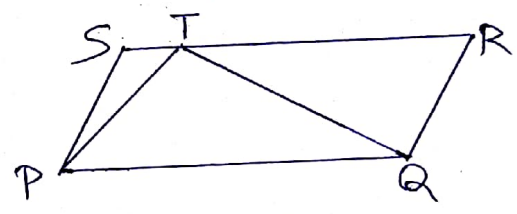
43) In  $\triangle ABC$ ,  $DE \parallel BC$  and  $D$  is mid-point of  $AB$ .  $AC = 16\text{cm}$ . Find  $AE$  and justify your answer.



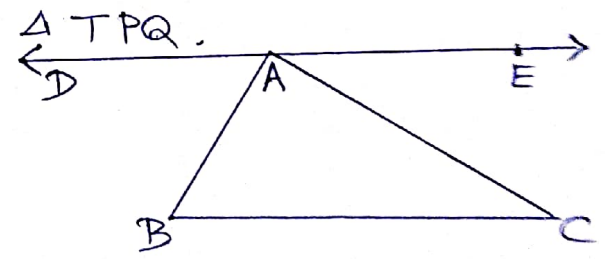
44) In the parallelogram  $ABCD$ , if  $\angle A = 45^\circ$  then find  $\angle C$  and  $\angle D$ . Justify.



45) In the figure,  $PQRS$  is a parallelogram and its area is  $212\text{cm}^2$ . Find the area of  $\triangle TPQ$ .



46)  $DE \parallel BC$  in  $\triangle ABC$ . If  $\angle DAB = 52^\circ$  and  $\angle BAC = 30^\circ$ , find  $\angle ABC$  and  $\angle ACB$ .

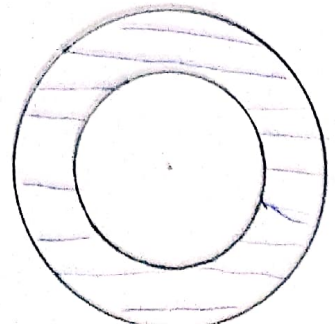


47) (i) If radius of a circle is  $7\text{cm}$ , find the area of the circle.

ii) If the radius of the above circle is doubled, then how much is the area of the circle?

48) If the diameter of a circle is  $42\text{cm}$ , find the perimeter of the corresponding semi-circle.

49) In a circular ring, the width of the ring is  $3\text{cm}$  and the outer radius is  $5\text{cm}$ . Find the area of ring.



- 50) If the ratio of radii of two circles are 3:15, find the ratio of their areas.
- 51) If the circumference of a circle is 44cm, find the radius of the circle.
- 52) Find the mean of first 10 natural numbers.
- 53) Find the median of first 14 natural numbers.
- 54) Find the range of 1st 14 natural numbers.
- 55)  $x\%$  of 400 is 28. Find  $x$ .

-x-